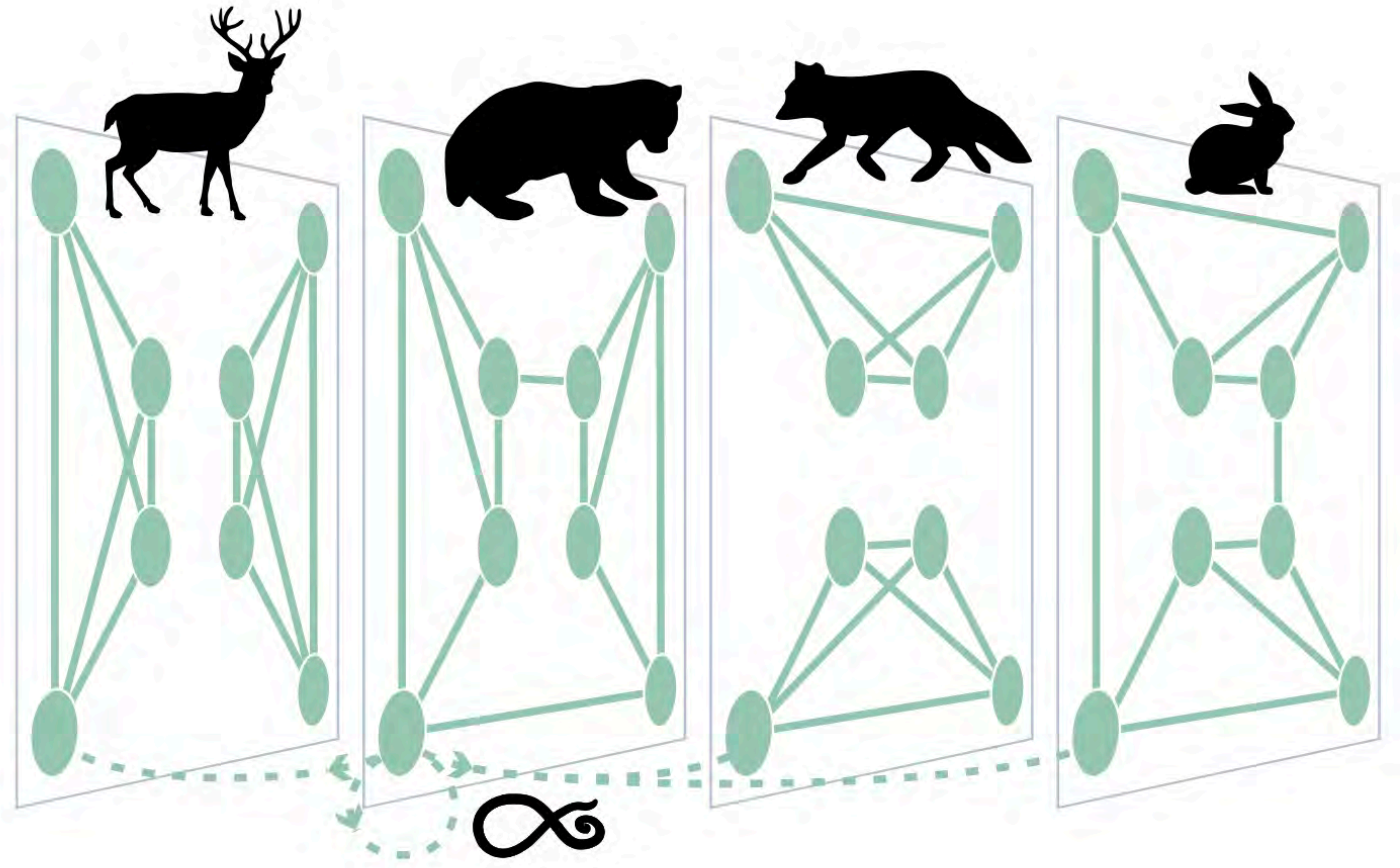
Multilayer networks

Analysis of Ecological - Biological Networks

Prof. Shai Pilosof



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Ben-Gurion University of the Negev

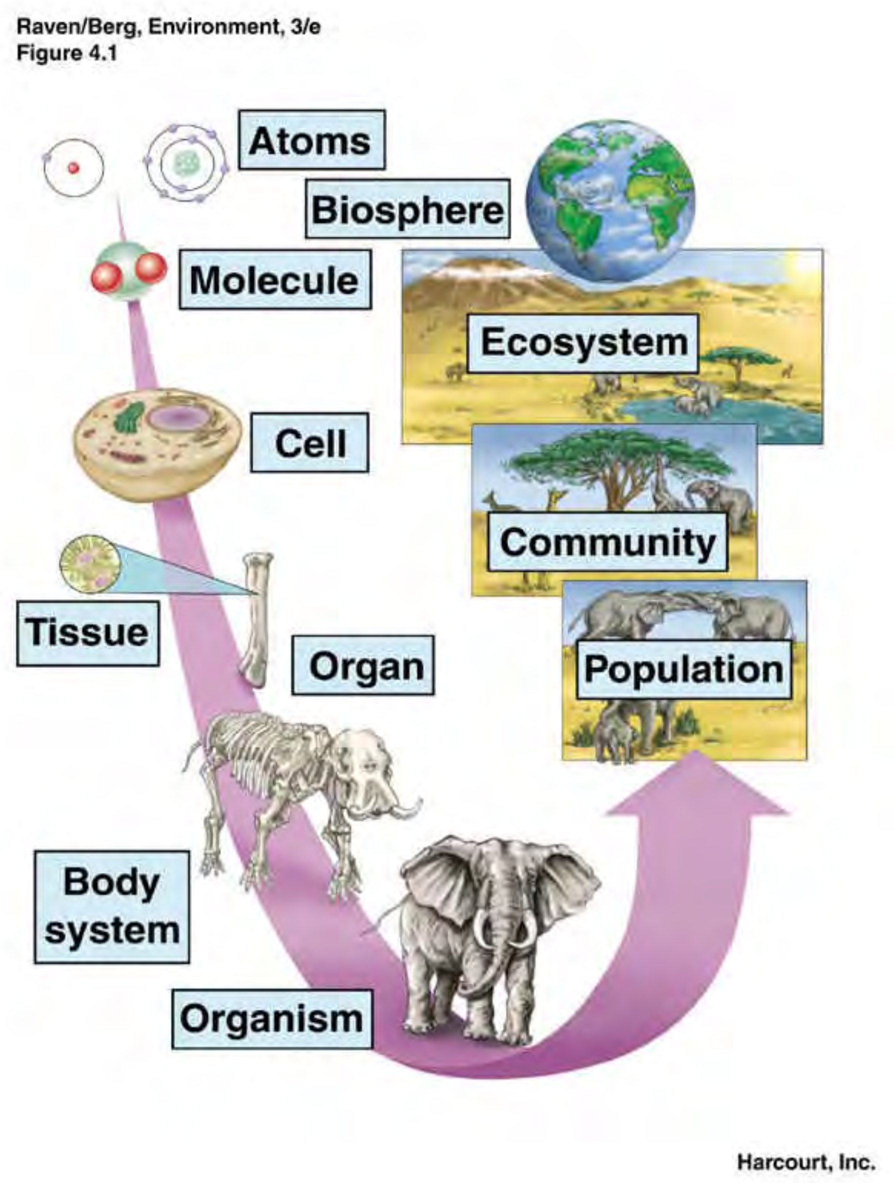
Class goals

1. Introduce multilayer networks.
2. Emphasize the role of interlayer edges.
3. Provide examples for research questions.

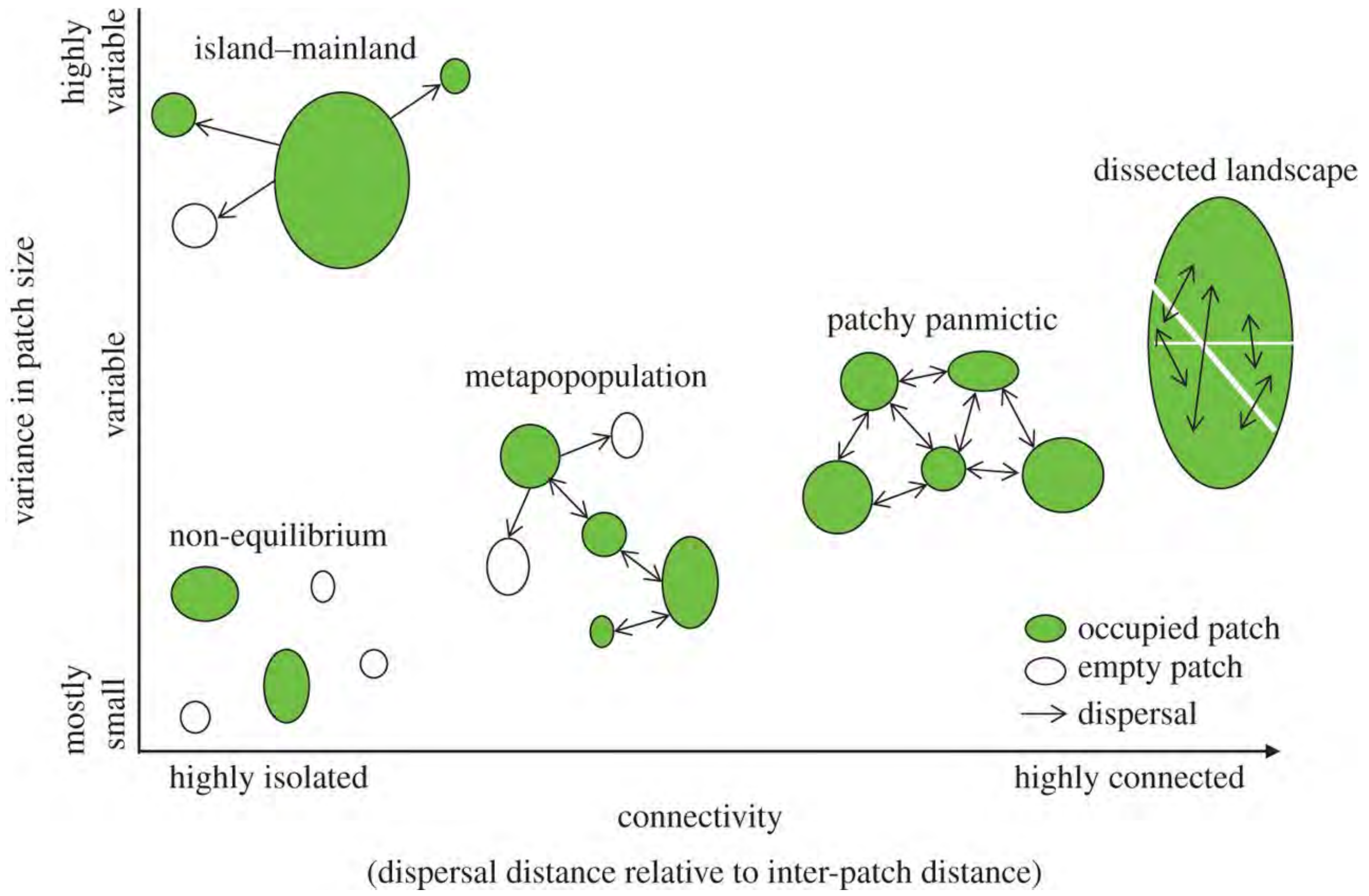
Outline

- History of multilayer networks in ecology.
- Definitions and tools.
- Interlayer links.
- Research directions (what can we study?)

Higher dimensionality in ecology

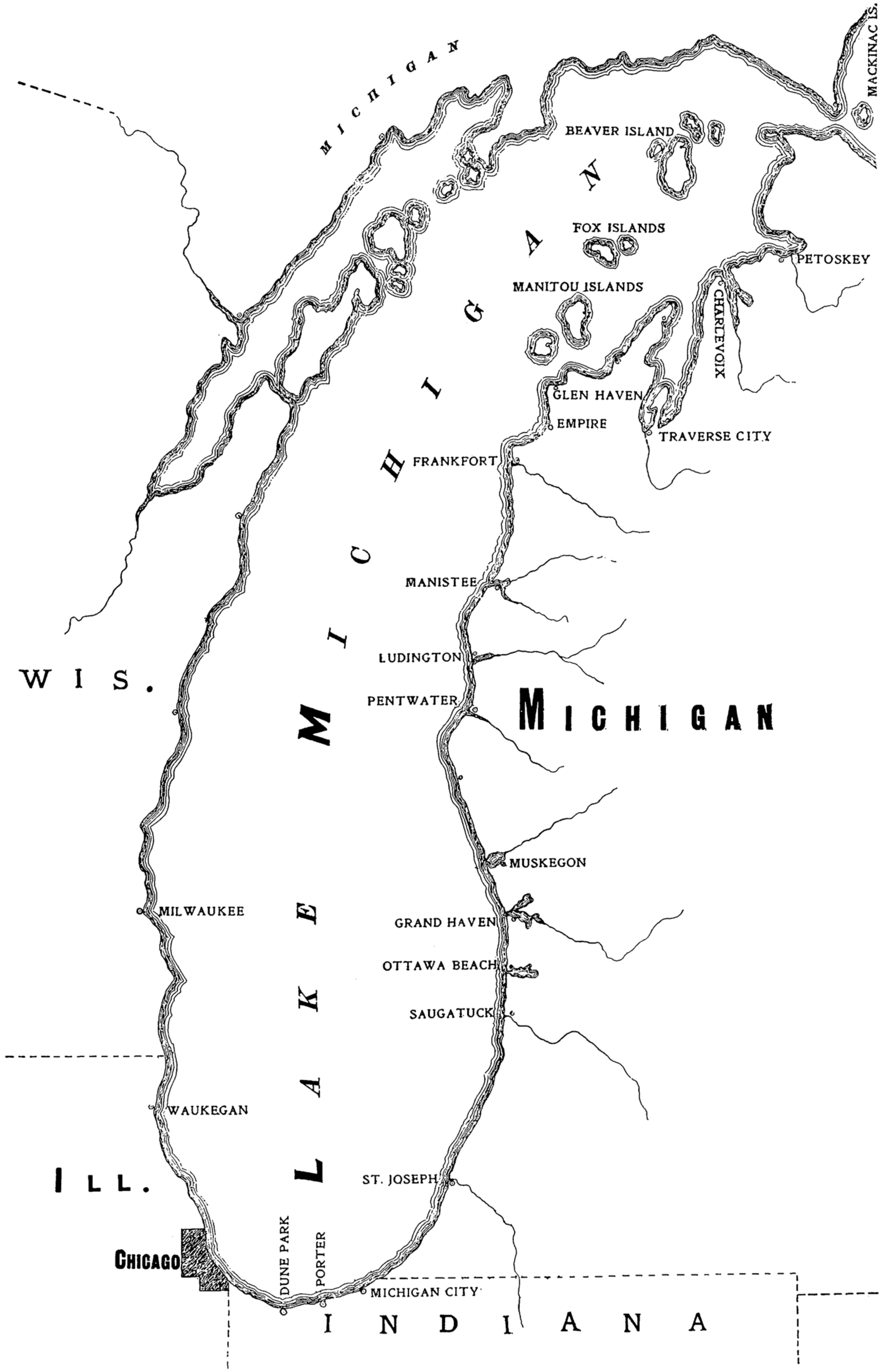


Organizational hierarchy



Space / time

Multiple kinds of interactions



"…so ecologists seek to study those plant structures which are changing at the **present time**,

and thus to throw light on the **origin of plant structures themselves.**"

Cowles HC. Botanical Gazette, 1899

Data

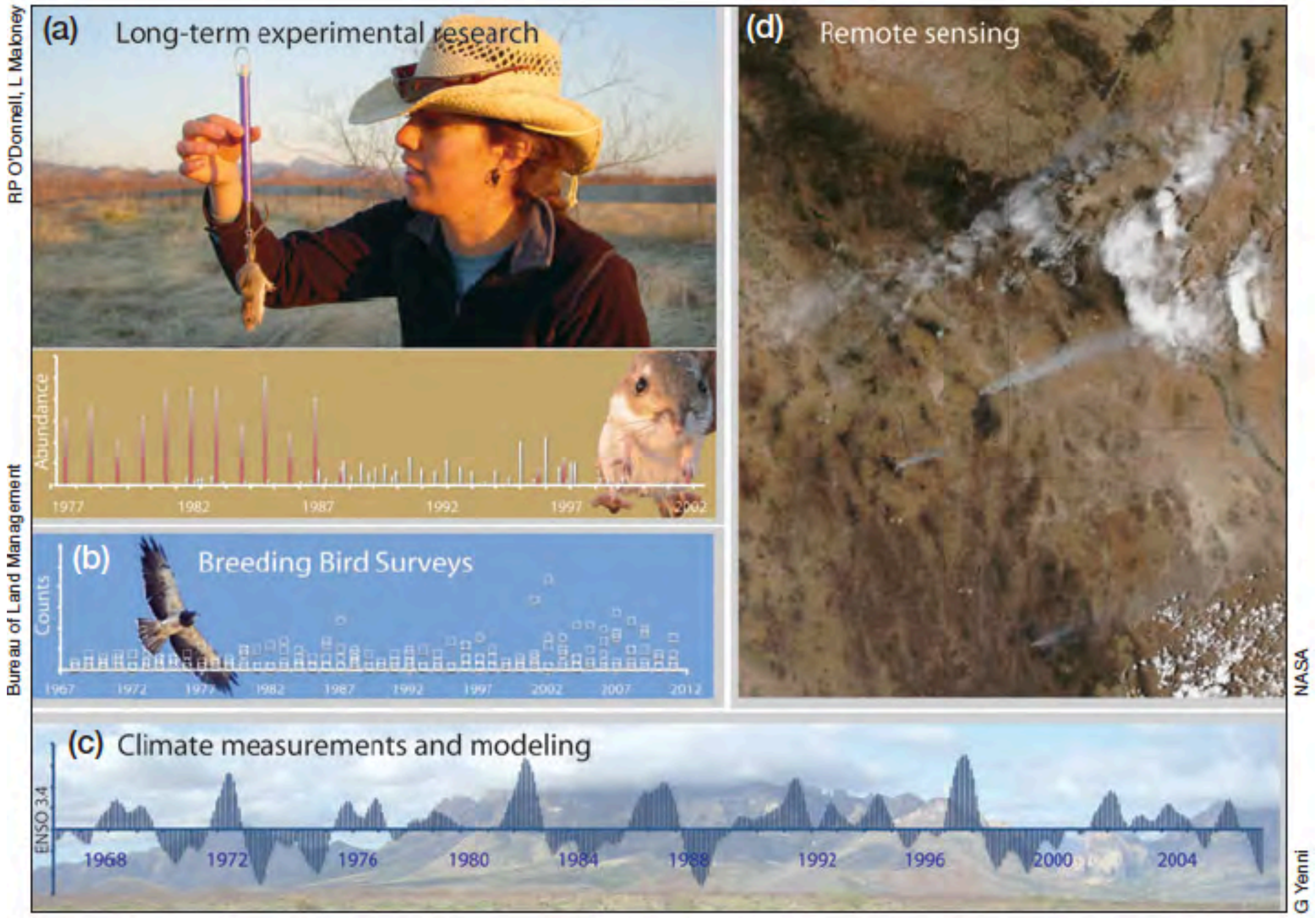


Figure from Hampton et al. 2013

Tools

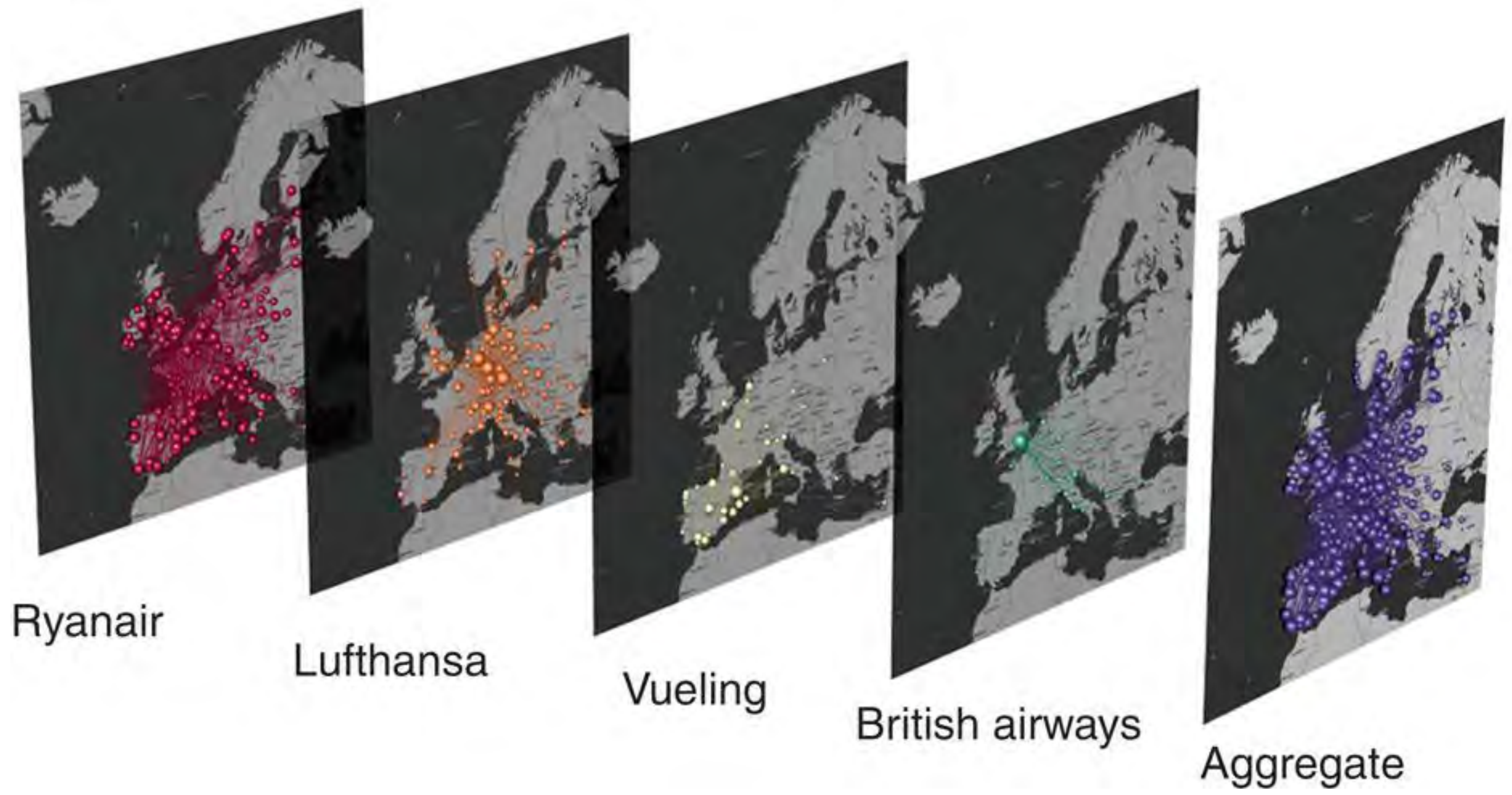


Figure from Cardillo et al. 2014

First explorations of (modern) ecological multilayer networks

Ecological Monographs, 60(3), 1990, pp. 331–367

SPATIAL AND TEMPORAL VARIATION IN TROPICAL FISH TROPHIC NETWORKS[1]

KIRK O. WINEMILLER
Department of Zoology and Texas Memorial Museum, The University of Texas, Austin, Texas 78712 USA

Ecological Monographs, 61(3), 1991, pp. 267–298

TEMPORAL VARIATION IN FOOD WEB STRUCTURE: 16 EMPIRICAL CASES[1]

KENNETH SCHOENLY AND JOEL E. COHEN
Rockefeller University, 1230 York Avenue, Box 20, New York, New York 10021-6399 USA

OIKOS 48: 280–288. Copenhagen 1987

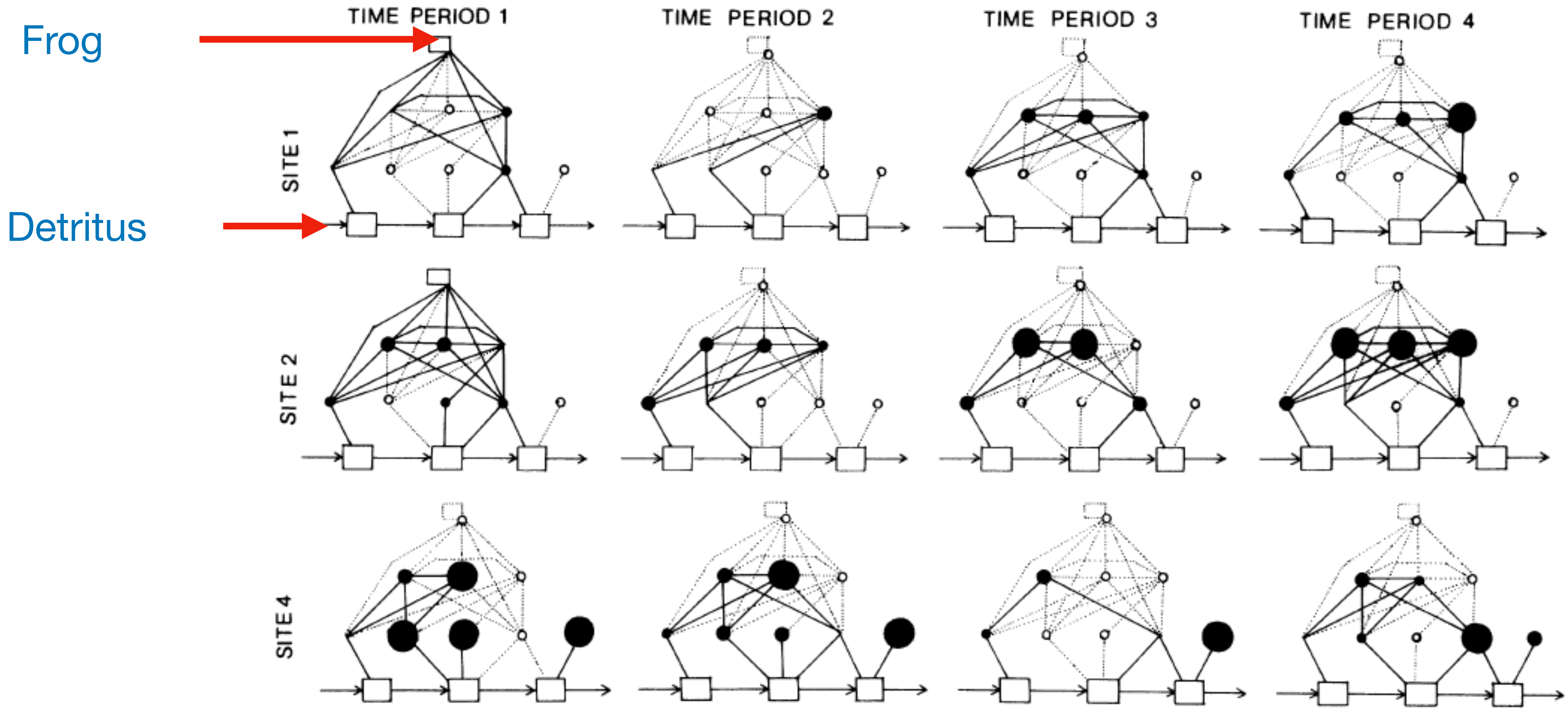
Spatial and temporal variation in food webs in water-filled treeholes

R. L. Kitching

OIKOS 55: 299–311. Copenhagen 1989

Spatial and temporal variation in the structure of a freshwater food web

Philip H. Warren



Kitching 1987 Oikos

Consumer (species number)

Resource (species number)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	22	23	24	25	27	28	29	30
0	·	1	1	2	·	2	1	·	·	·	·	·	·	2	2	2	2	2	2	2	·	·	·	·	·	·	·	1
1	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·
2	2	·	·	·	·	·	·	1	2	2	2	2	·	·	·	·	·	·	·	·	2	2	2	2	·	2	2	2
3	2	·	·	·	2	·	·	·	1	1	1	2	2	·	·	·	·	·	·	·	·	1	2	2	2	2	·	2
4	·	·	·	·	2	·	·	·	1	1	1	1	·	·	·	·	·	·	·	·	·	1b	2b	1b	2b	1	·	2
5	·	·	·	·	·	·	·	·	·	·	2	2	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·
6	·	·	·	·	2	·	·	·	1	1	1	2	2	·	·	·	·	·	·	·	2	2	2	1	·	2	1	·
7	·	·	·	·	1	·	·	2	1	1	1	1	1a	·	·	·	·	·	·	·	2	·	·	·	·	1	1	·
8	·	·	·	·	2	·	·	2	1	1	1	1	2	·	·	·	·	·	·	·	·	2	2	2	·	1	1	·
9	·	·	·	·	·	·	·	·	1	1	1	1	1	·	·	·	·	·	·	·	·	1	2	2	2	·	·	·
10	·	·	·	·	·	·	·	·	2	1	1	2	2	·	·	·	·	·	·	·	·	1	2	2	2	·	·	·
11	·	·	·	·	·	·	·	·	·	·	1	·	·	·	·	·	·	·	·	·	·	·	·	·	2a	·	·	·
12	·	·	·	·	·	·	·	·	·	·	2a	·	·	·	·	·	·	·	·	·	·	·	·	·	2a	·	·	·
13	·	·	·	·	·	·	·	·	2c	2c	2c	2c	2	·	·	·	·	·	·	·	·	2c	2c	2c	2c	·	·	·
14	·	·	·	·	1	·	·	·	1c	1c	1c	1c	2c	·	·	·	·	·	·	·	·	1c	2c	1c	2c	1c	1c	·
15	·	·	·	·	2c	·	·	·	1c	1c	1c	1c	2c	·	·	·	·	·	·	·	·	1c	1c	1c	2c	1c	1c	·
16	·	·	·	·	2c	·	·	·	1c	1c	1c	1c	2c	·	·	·	·	·	·	·	·	1c	1c	1c	2c	1c	1c	·
17	·	·	·	·	2c	·	·	·	1c	1c	1c	1c	2c	·	·	·	·	·	·	·	·	1c	2c	1c	2c	1c	1c	·
18	·	·	·	·	2c	·	·	·	1c	1c	1c	1c	2c	·	·	·	·	·	·	·	·	1c	2c	1c	2c	1c	1c	·
19	·	·	·	·	2c	·	·	·	1c	1c	1c	1c	2c	·	·	·	·	·	·	·	·	1c	2c	1c	2c	1c	1c	·
20	·	·	·	·	2c	·	·	·	1c	1c	1c	1	2c	·	·	·	·	·	·	·	·	1c	2c	1c	2c	1c	1c	·
22	·	·	·	·	·	·	·	·	2c	2c	2	2	1c	·	·	·	·	·	·	·	2c	2c	2c	2c	2c	2c	2c	·
23	·	·	·	·	·	·	·	·	1c	1c	1c	1c	1c	·	·	·	·	·	·	·	·	2c	2c	2c	2c	·	·	·
24	·	·	·	·	·	·	·	·	1c	1c	1c	1c	1c	·	·	·	·	·	·	·	·	2c	2c	2c	2c	·	·	·
25	·	·	·	·	·	·	·	·	1c	1c	1c	1c	1c	·	·	·	·	·	·	·	·	2c	2c	2c	2c	·	·	·
27	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	2c	·	·	·
28	·	·	·	·	·	·	·	·	1	1	1	1	2	·	·	·	·	·	·	·	2a	1	2	2	2	1	·	·
29	·	·	·	·	1	·	·	·	·	·	1	·	2	·	·	·	·	·	·	·	·	1	2	1	2	·	·	·
30	·	·	·	·	1	·	·	·	·	·	1	·	2	·	·	·	·	·	·	·	·	1a	2a	2a	2a	·	·	·

Warren 1989 Oikos

Consumer as adult

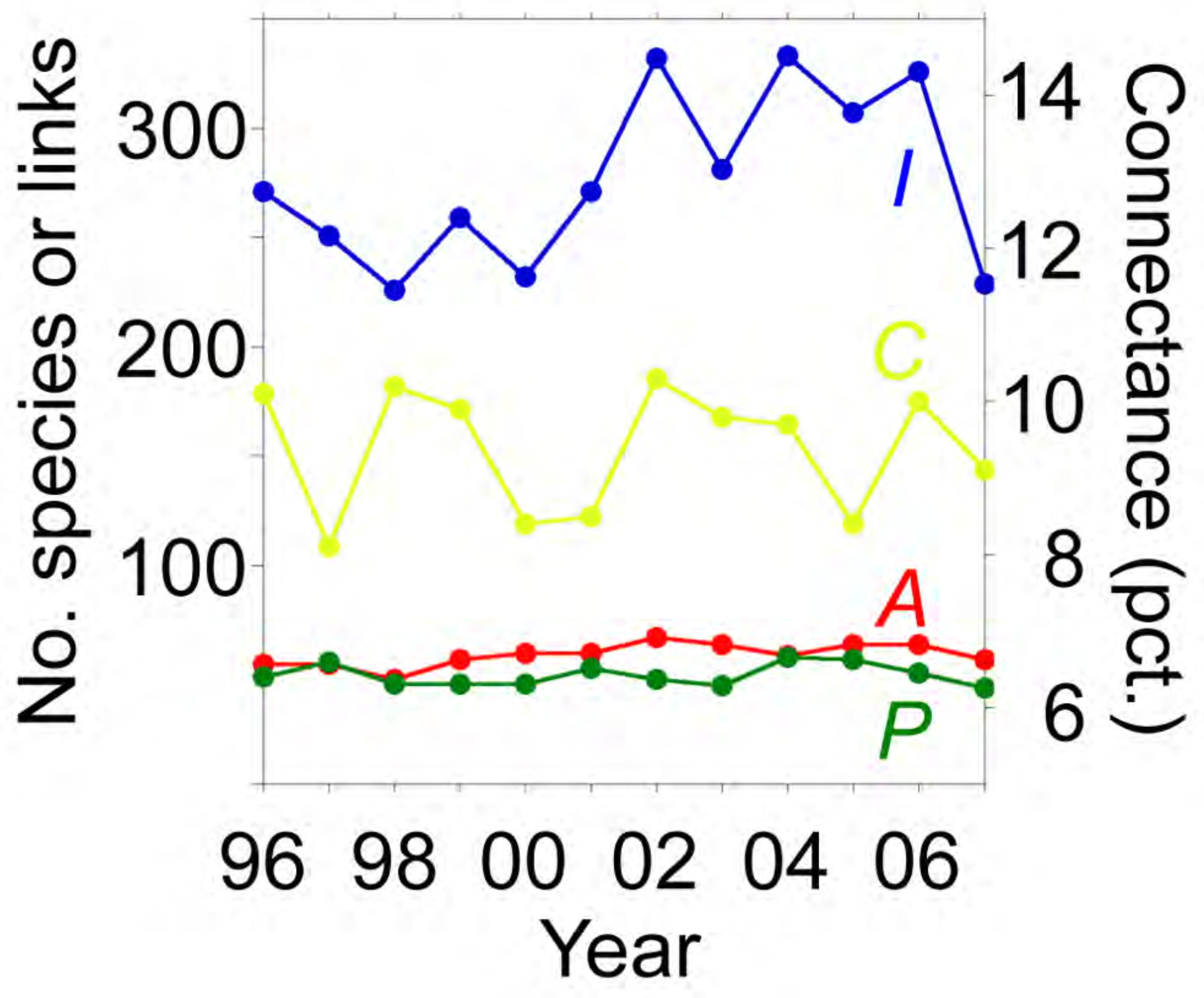
Consumer as larva

TEMPORAL VARIATION IN FOOD WEB STRUCTURE: 16 EMPIRICAL CASES[1]

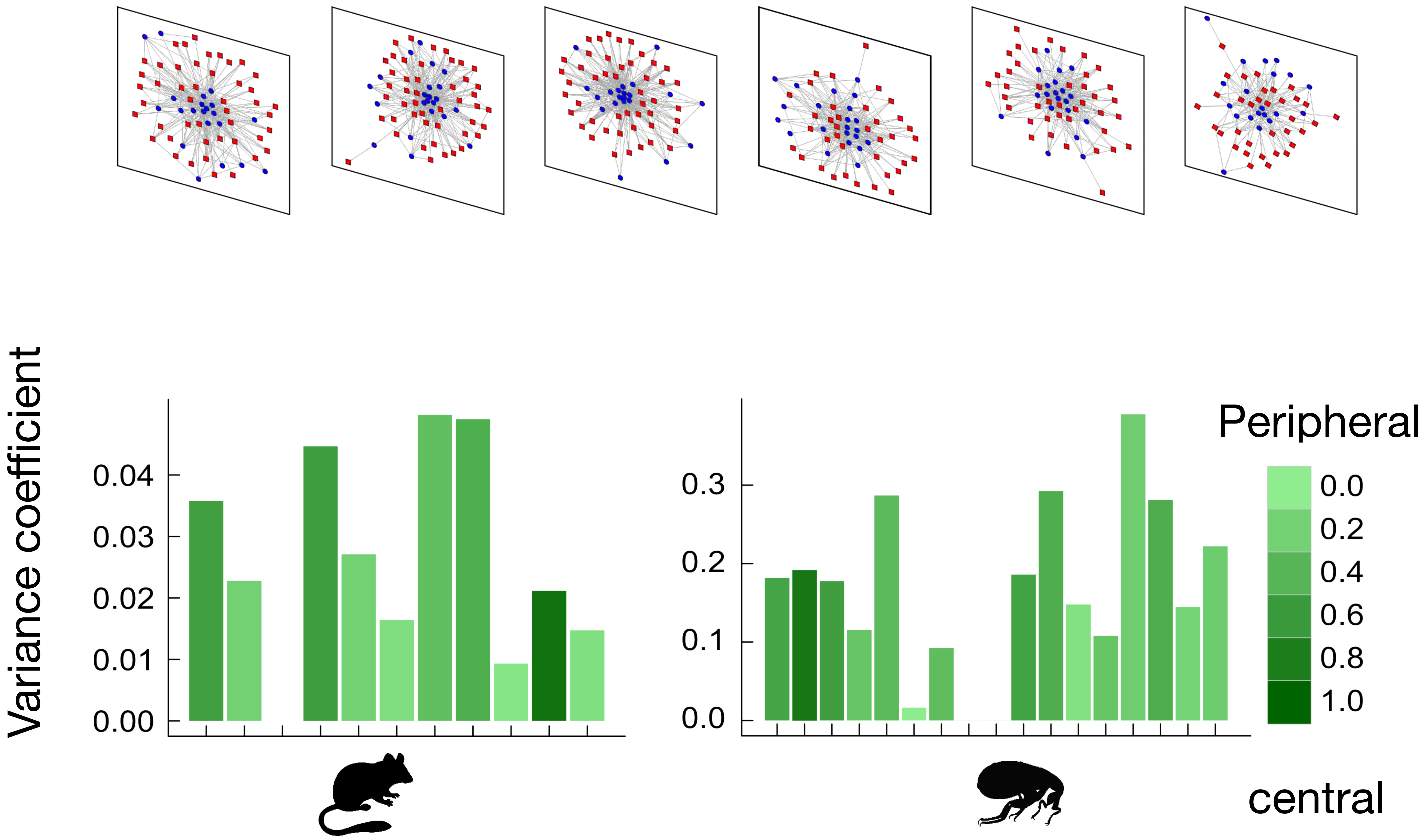
KENNETH SCHOENLY AND JOEL E. COHEN
Rockefeller University, 1230 York Avenue, Box 20, New York, New York 10021-6399 USA

Used 16 webs from **published** data

1. Quantify 9 properties.
2. Compare aggregated to time-specific.
3. Remove rare species to detect their effects on aggregation.

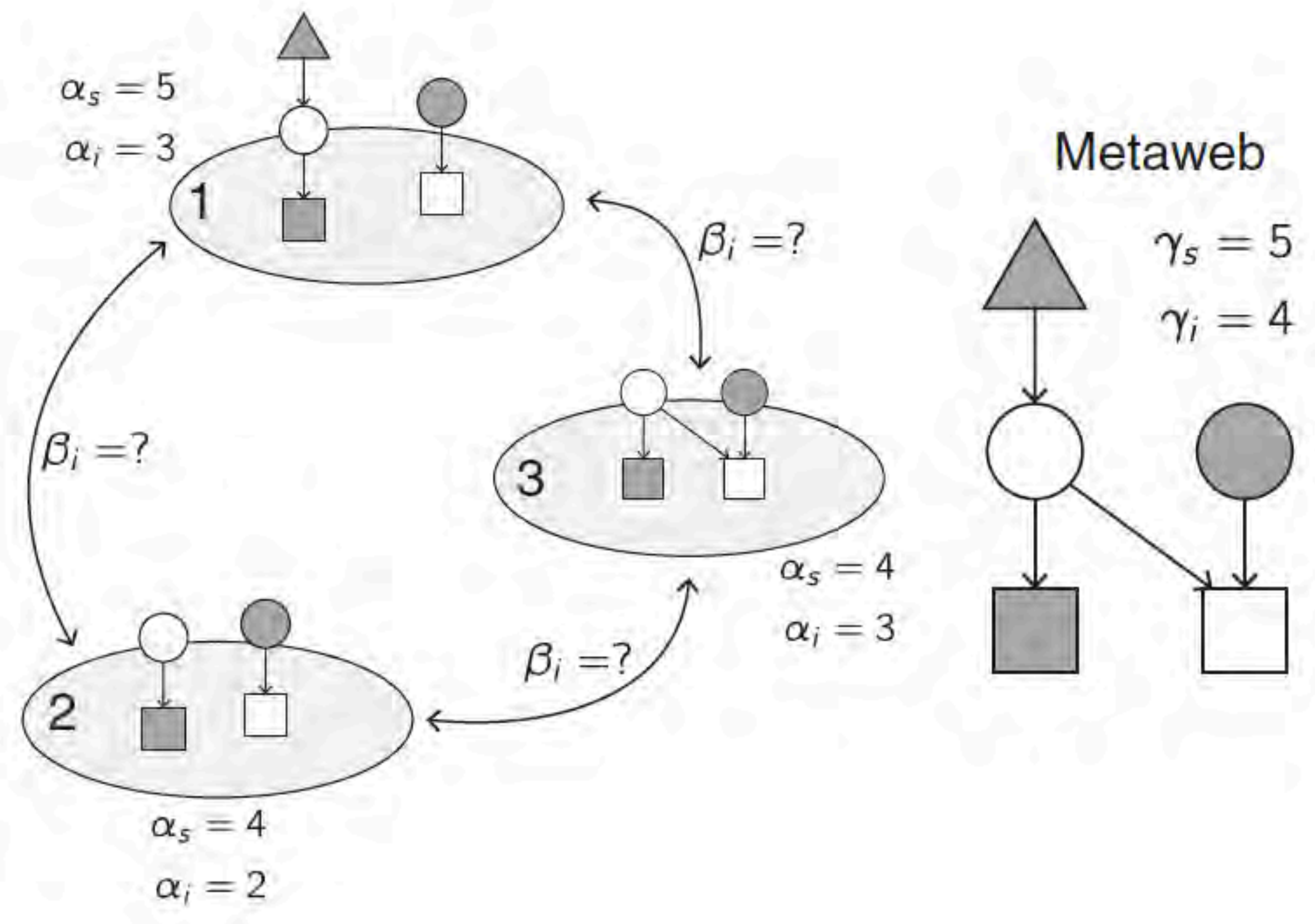


Olesen et al 2011, PLoS One



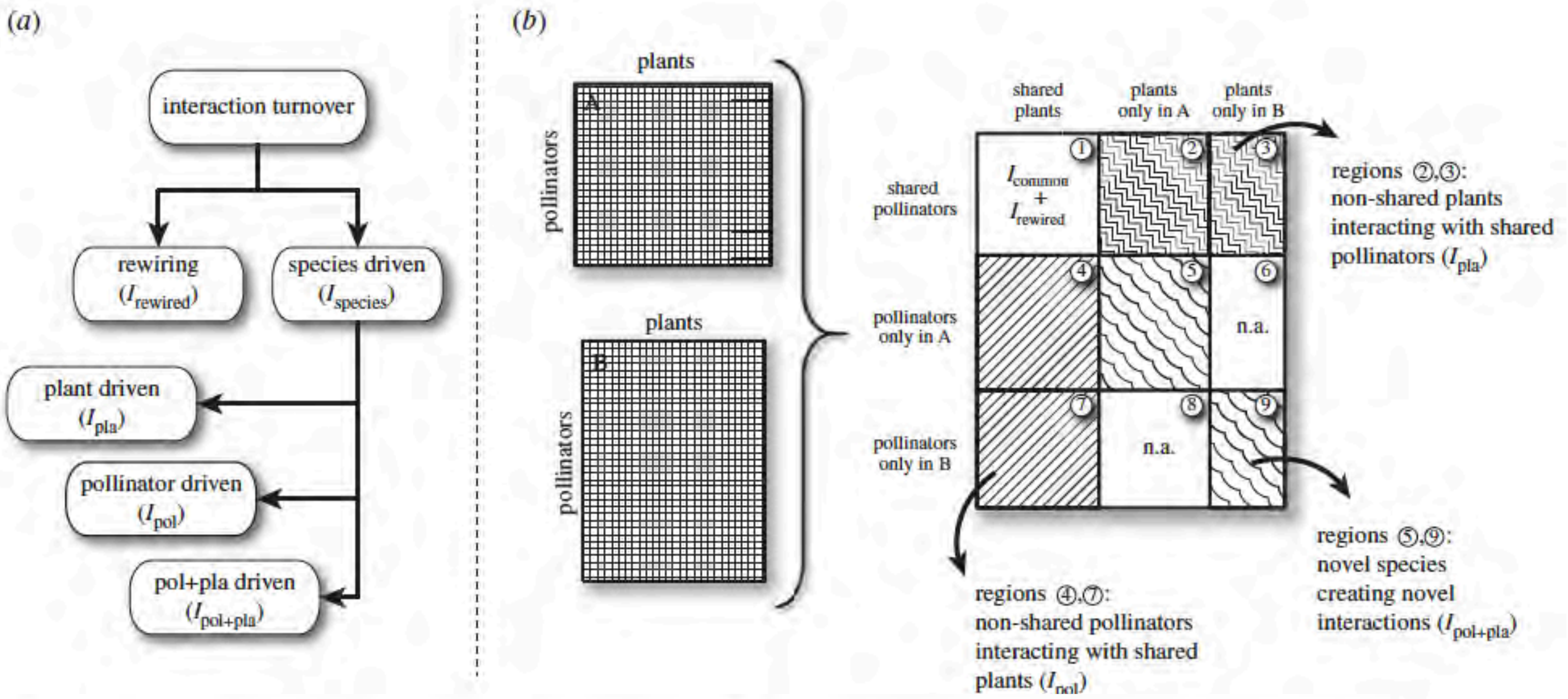
Pilosof et al 2013, J. Anim. Ecol.

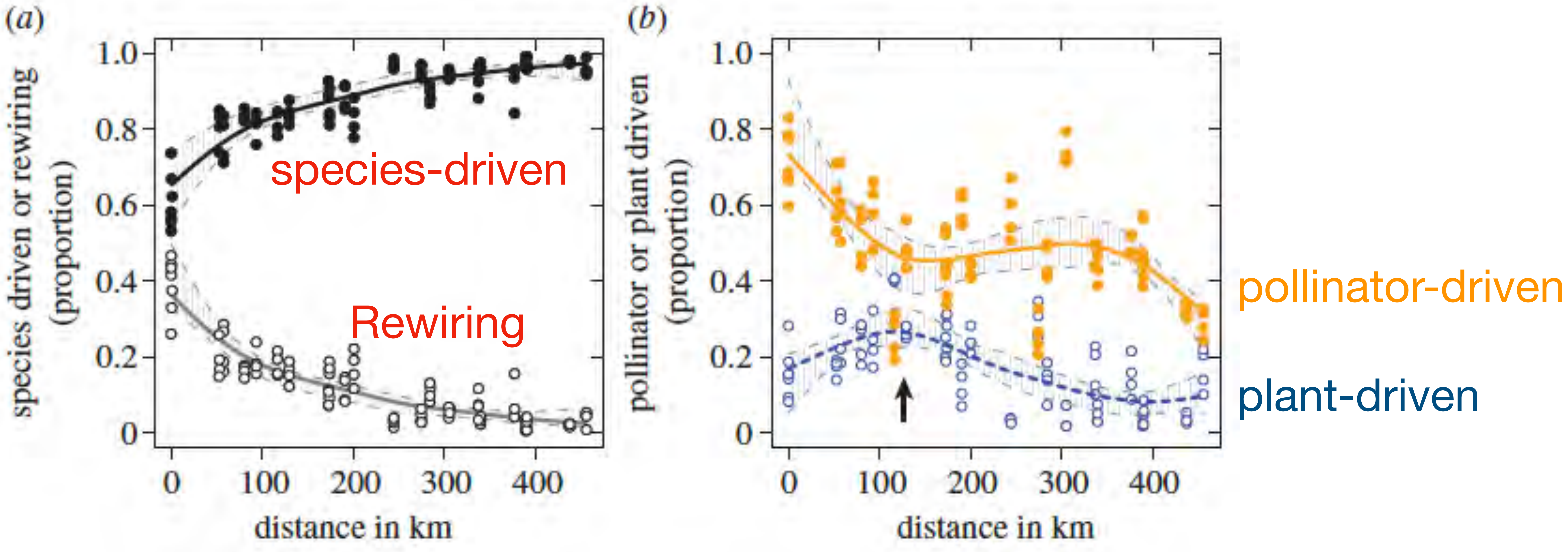
Species and interaction turnover in space and time



Measure	Definition
β_S	Dissimilarity in the species composition of communities
β_{OS}	Dissimilarity of interactions established between species common to both realisations
β_{WN}	Dissimilarity of interactions
β_{ST}	Dissimilarity of interactions due to species turnover
β'_{OS}	Dissimilarity between a local network and its counterpart in the metaweb
β_{ST}/β_{WN}	Contribution of species dissimilarity to network dissimilarity

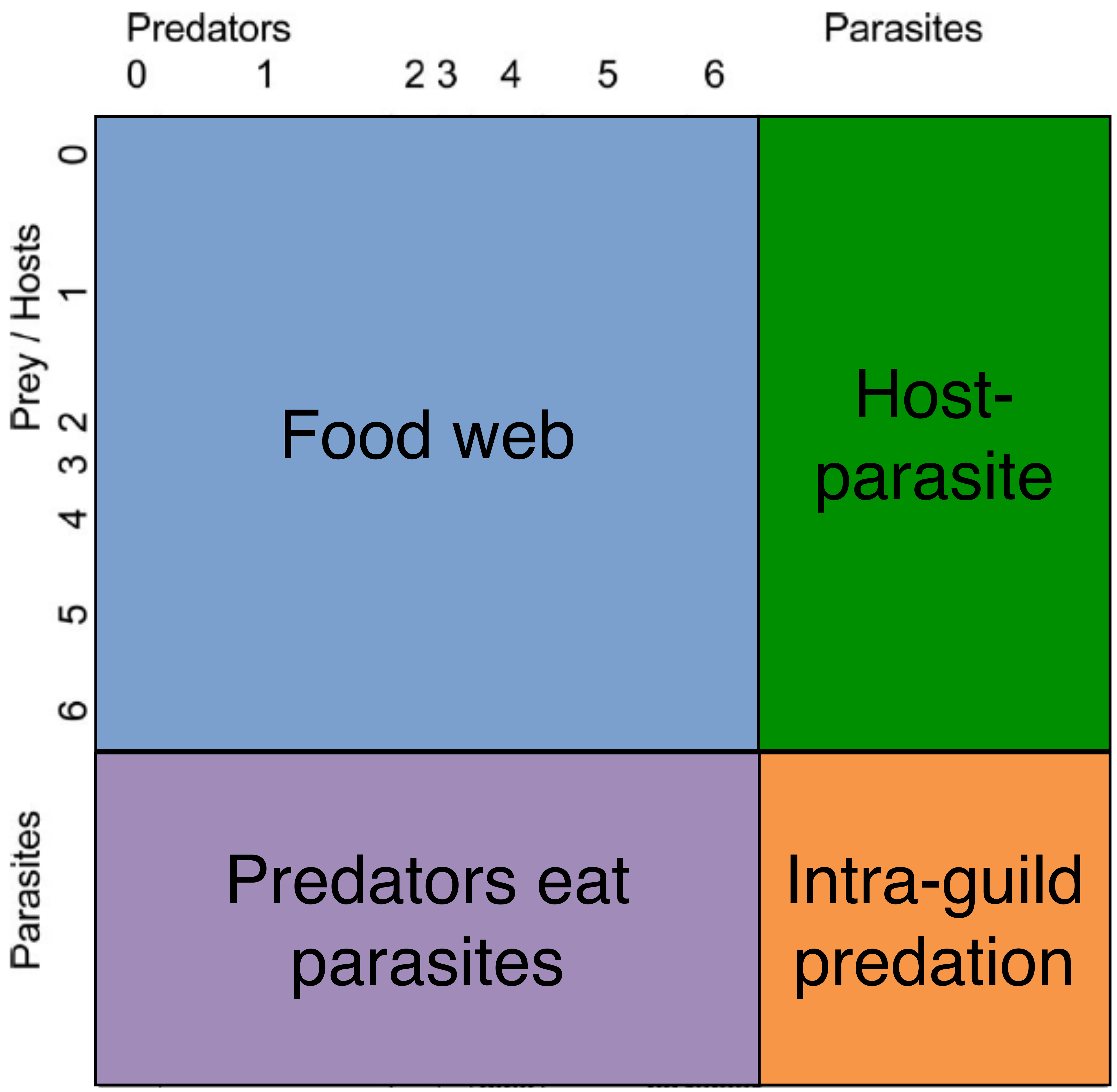
Poisot et al. 2012, Ecol. Lett.





Trøjelsgaard et al., 2015

Merging multiple kinds of interactions



Including parasites changes the structure of the food web

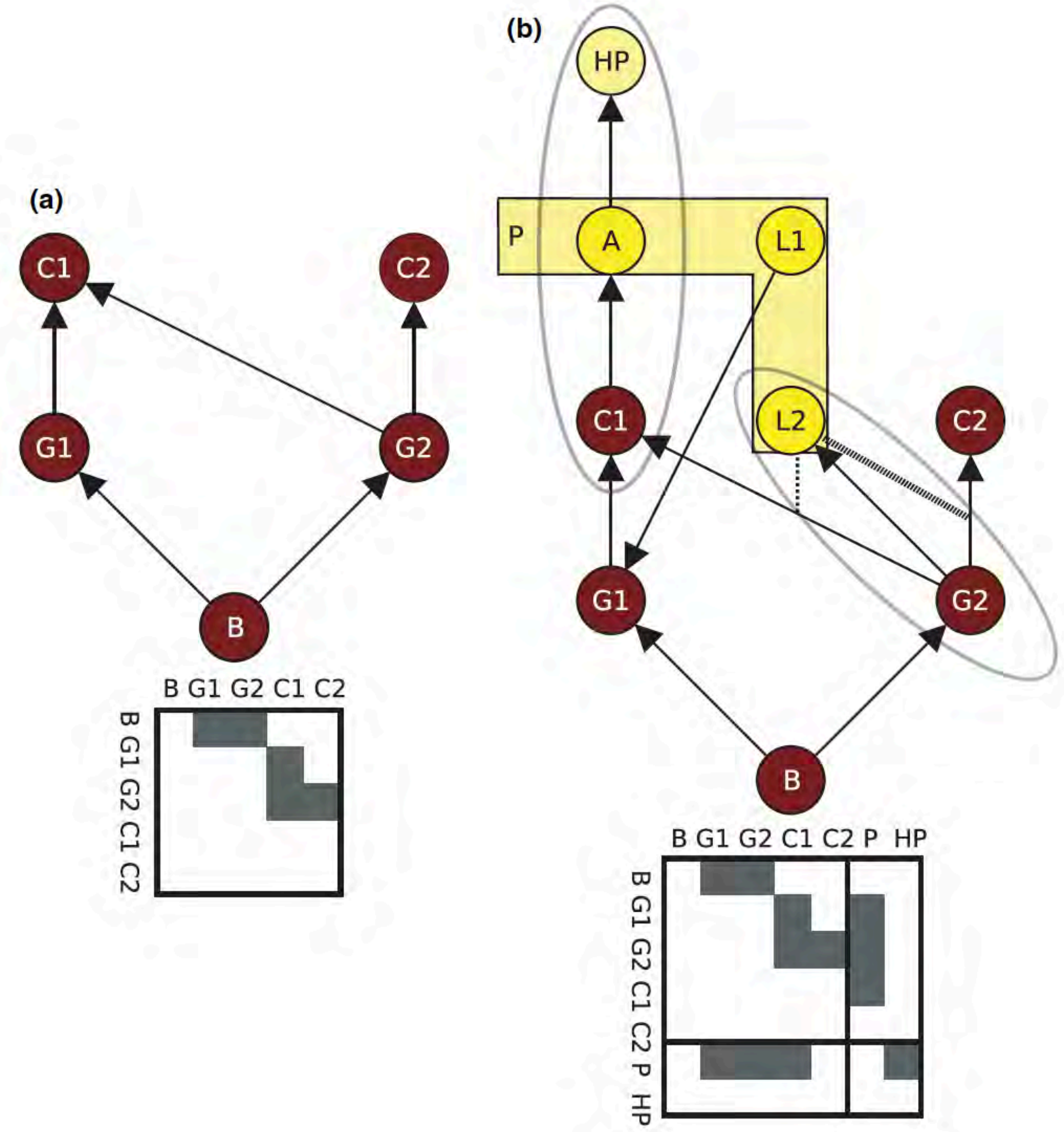
How to include parasites?
Multiple life stages?

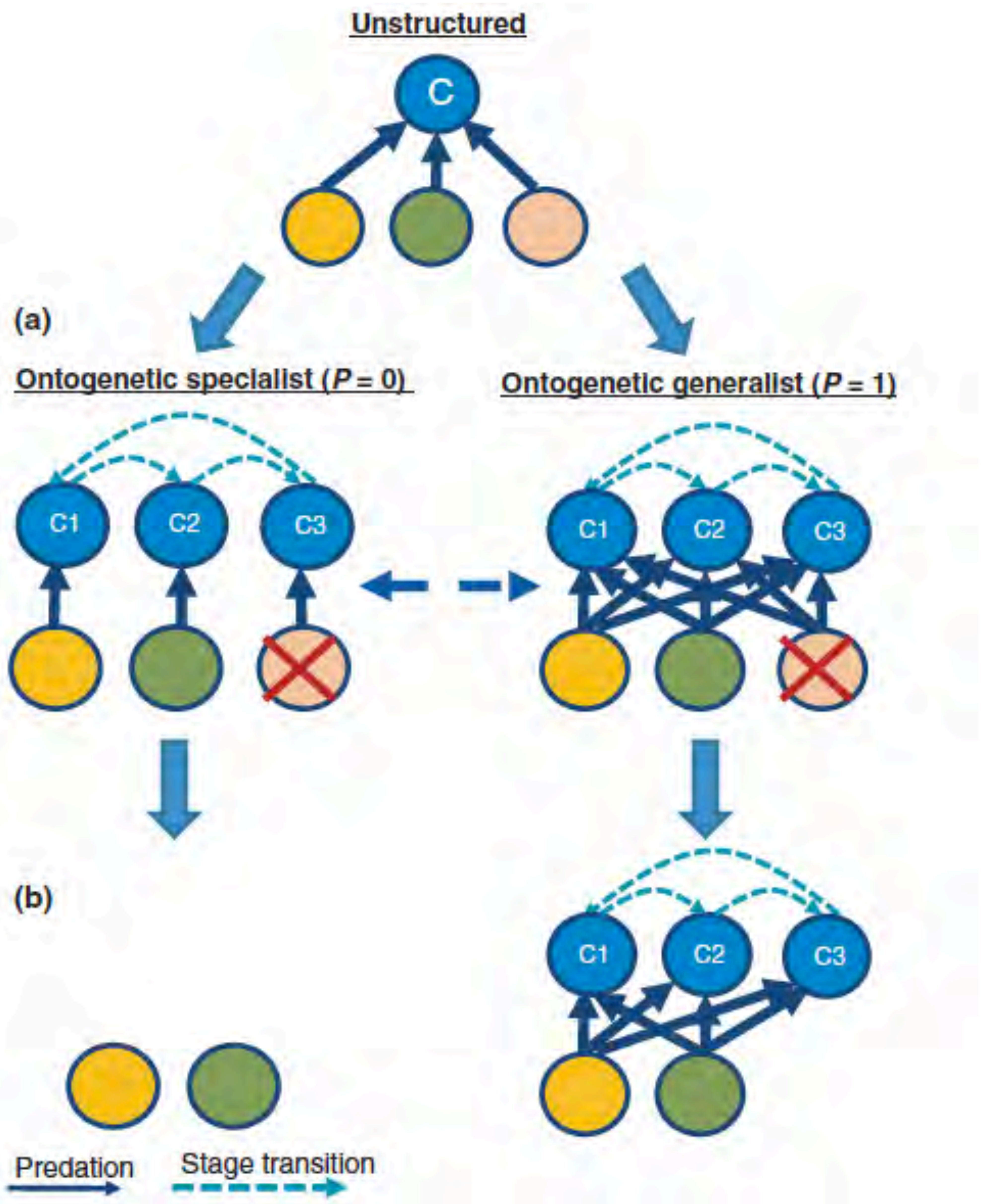
Lafferty, Dobson, Kuris. PNAS 2006

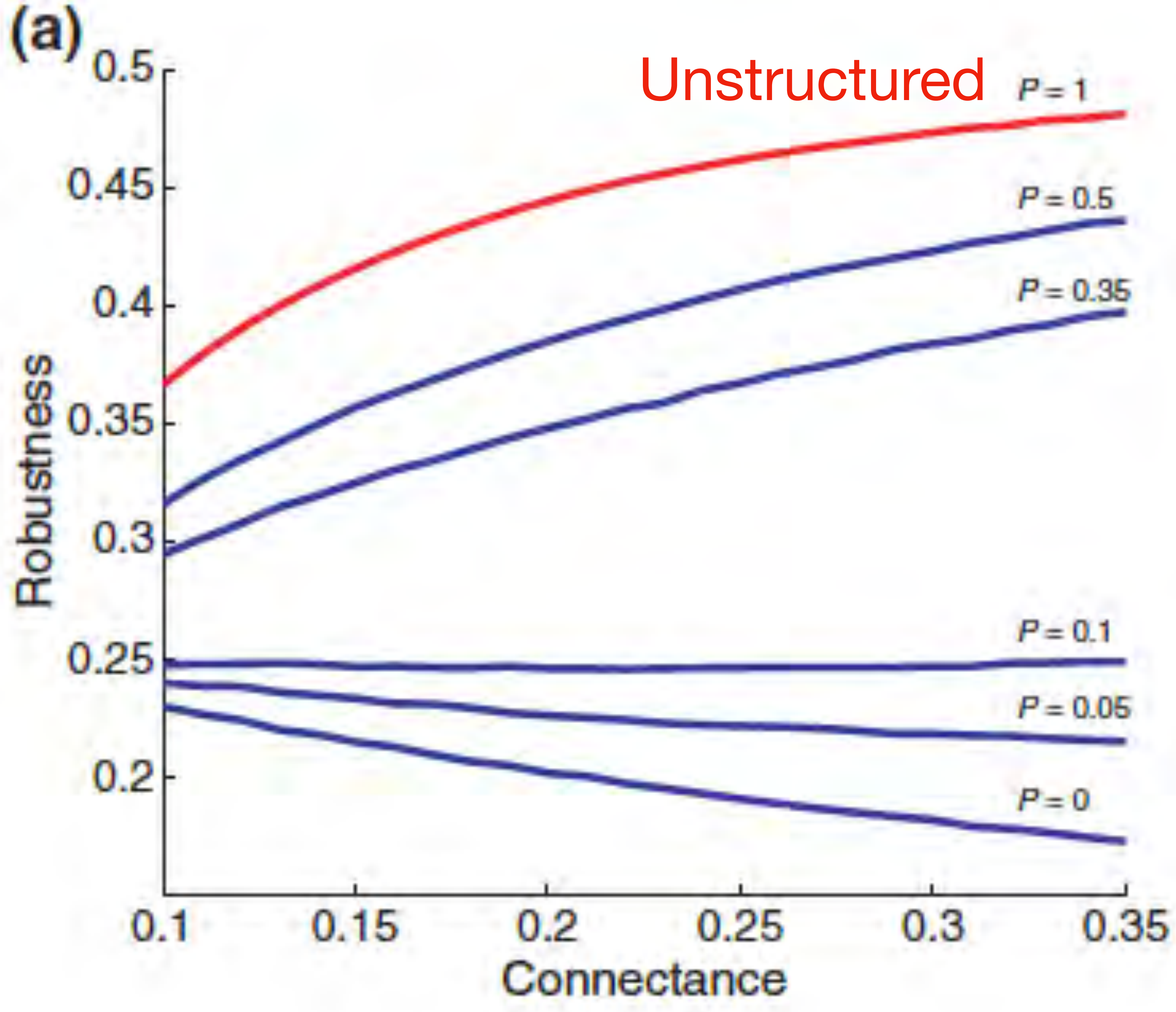
IDEA AND PERSPECTIVE

Parasites in food webs: the ultimate missing links

Kevin D. Lafferty,[1]* Stefano Allesina,[2] Matias Arim,[3,4] Cherie J. Briggs,[5] Giulio De Leo,[6] Andrew P. Dobson,[7] Jennifer A. Dunne,[8,9] Pieter T. J. Johnson,[10] Armand M. Kuris,[5] David J. Marcogliese,[11] Neo D. Martinez,[2,9] Jane Memmott,[12] Pablo A. Marquet,[4,13,14] John P. McLaughlin,[5] Erin A. Mordecai,[5] Mercedes Pascual,[14] Robert Poulin[15] and David W. Thieltges[15]







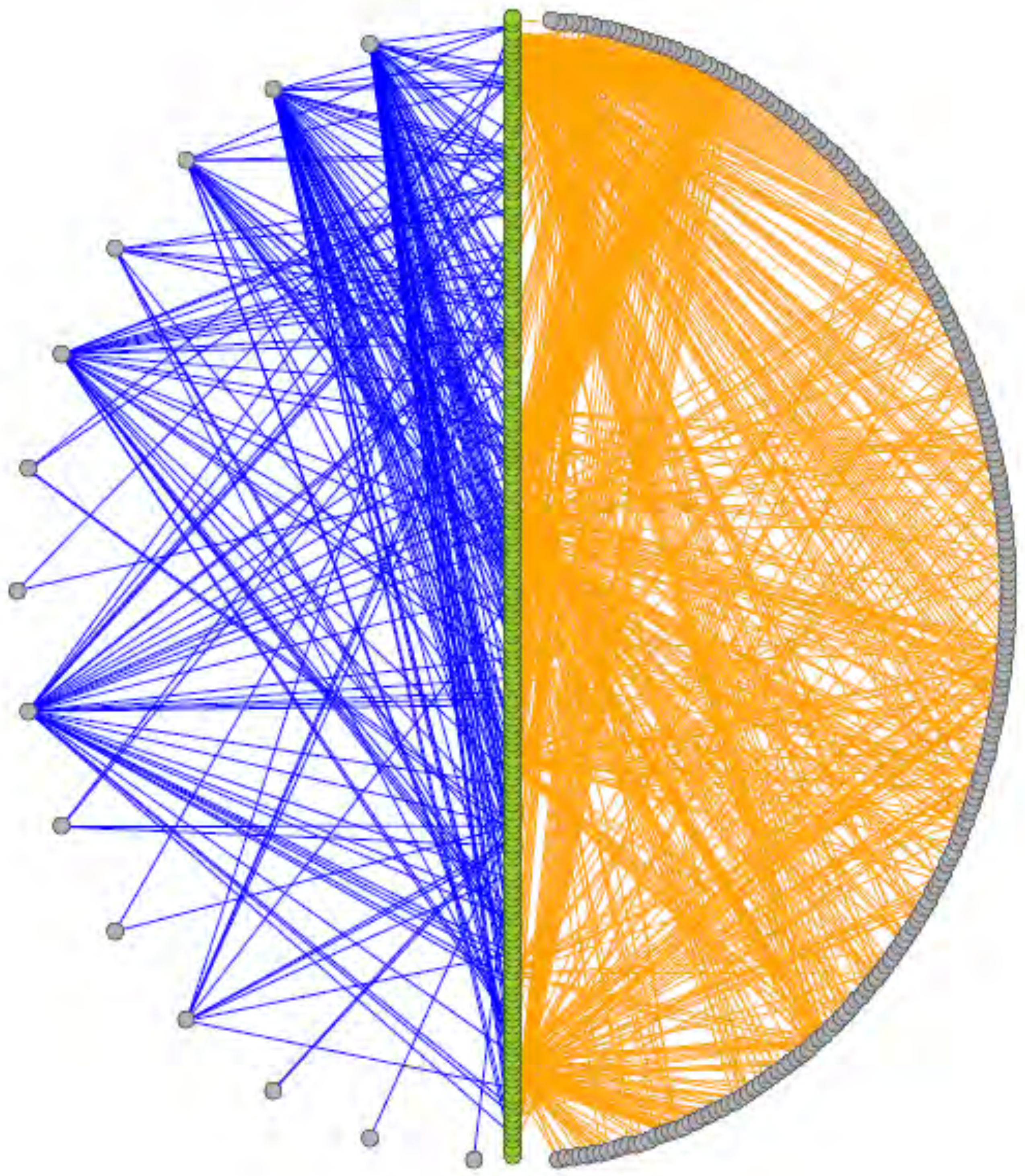
Rudolf and Lafferty 2011 Ecol. Lett

Oikos 118: 122–130, 2009
doi: 10.1111/j.1600-0706.2008.16751.x,

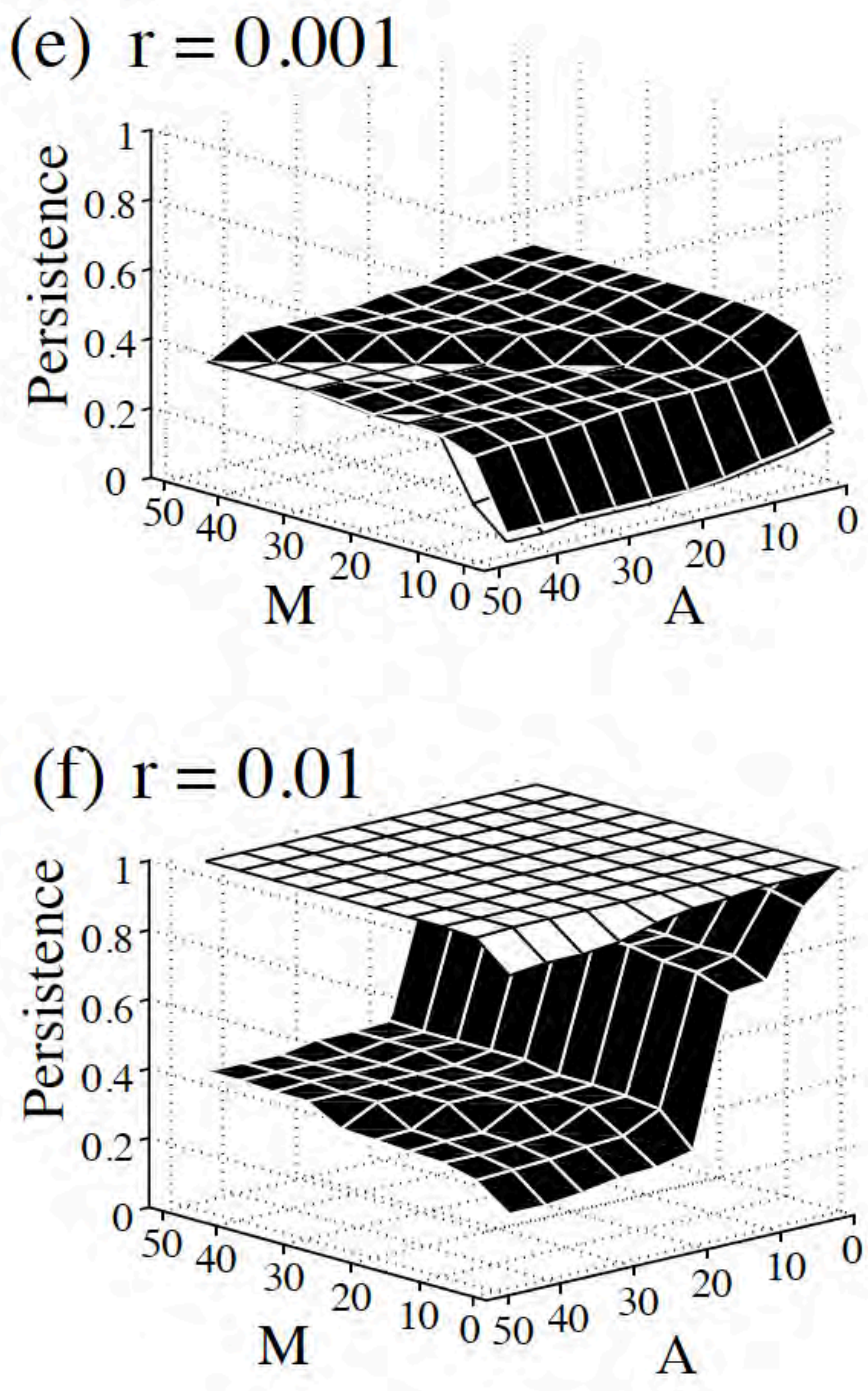
Subject Editor: Jonathan Shurin. Accepted 4 July 2008

Diversity in a complex ecological network with two interaction types

Antagonistic Plants Mutualistic



Doñana Biological Reserve, Spain

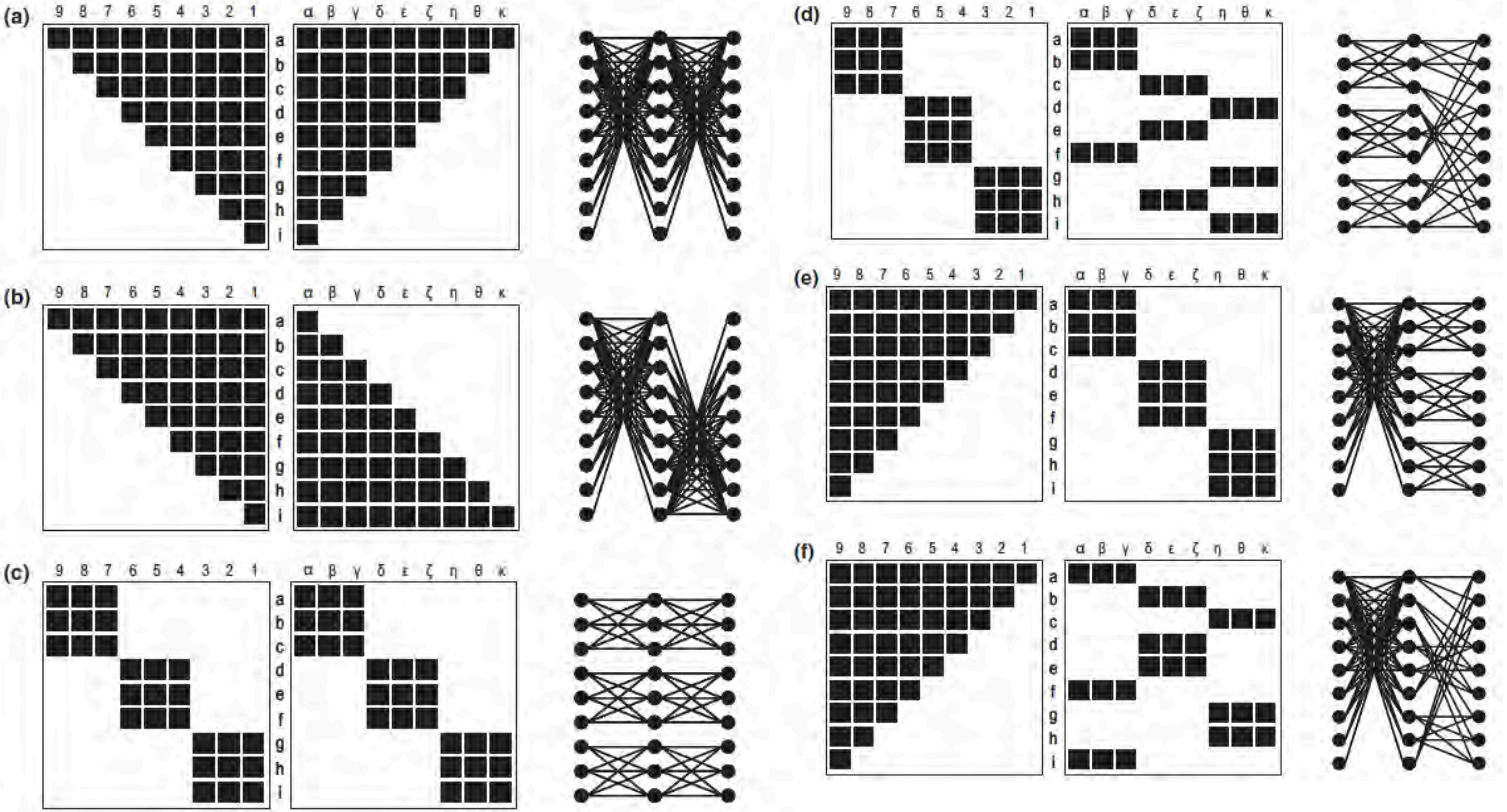


Ecology Letters, (2011) doi: 10.1111/j.1461-0248.2011.01688.x

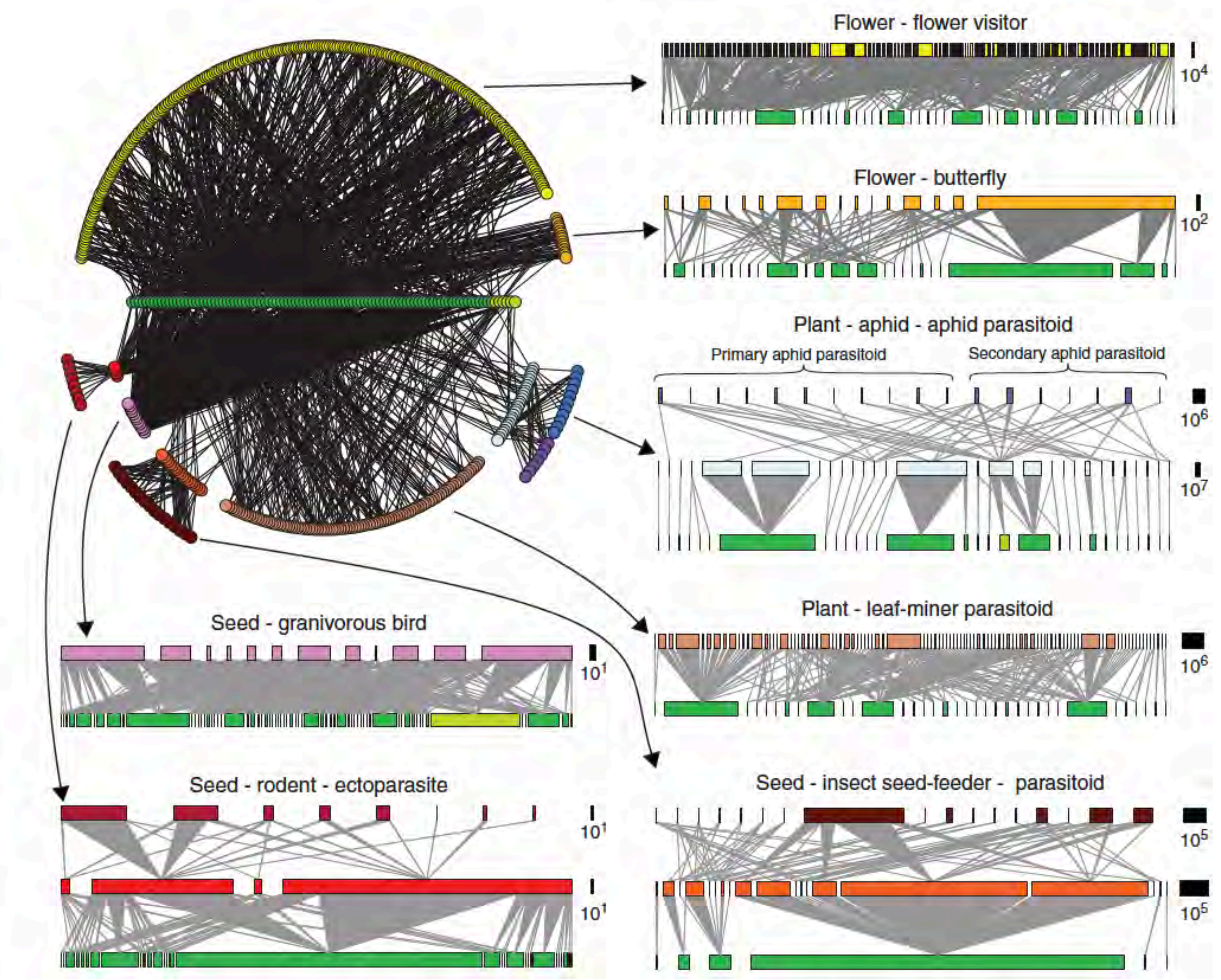
REVIEW AND SYNTHESIS

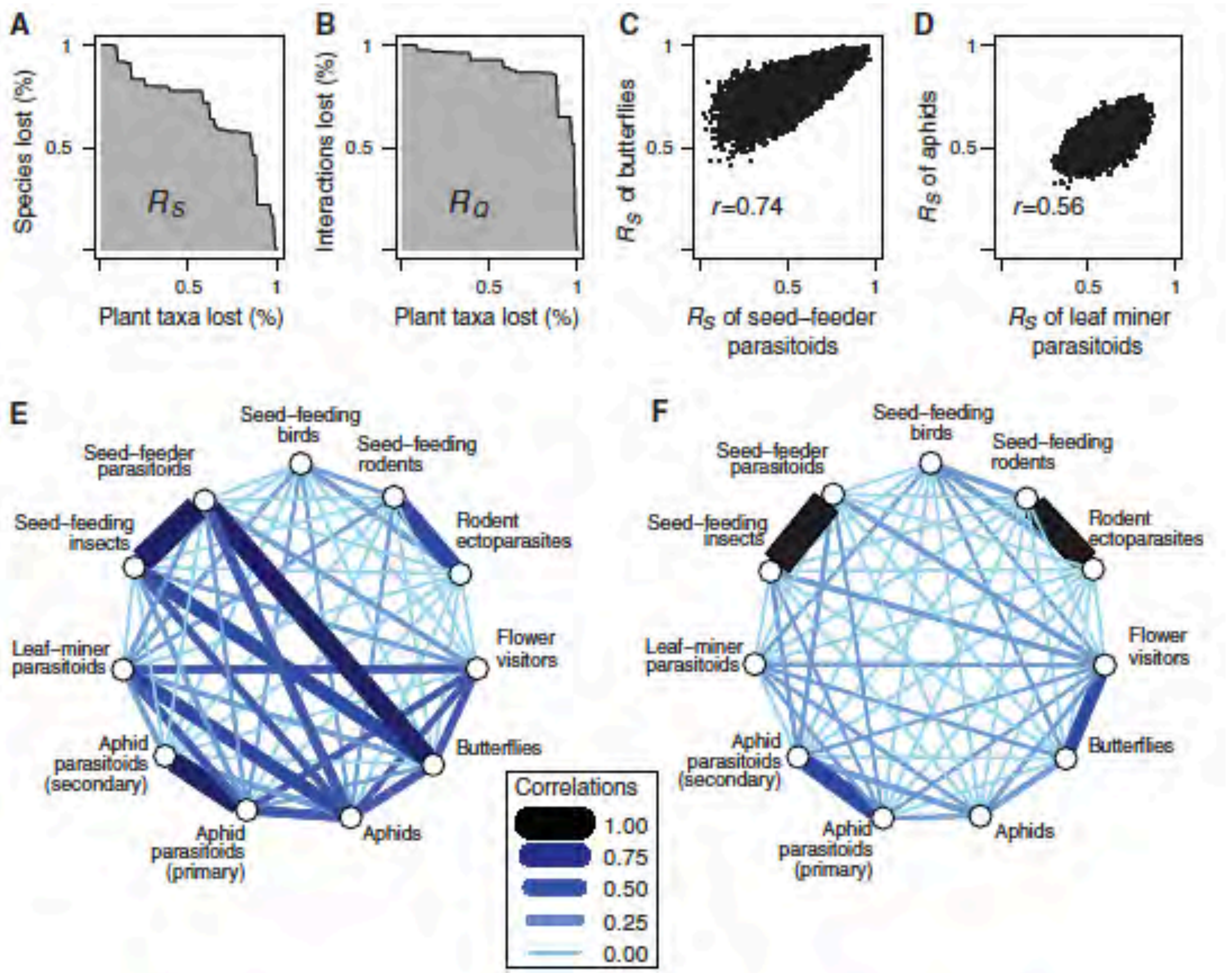
The ecological and evolutionary implications of merging different types of networks

Colin Fontaine,[1]* Paulo R. Guimarães Jr,[2] Sonia Kéfi,[3] Nicolas Loeuille,[4,5] Jane Memmott,[6] Wim H van der Putten,[7,8] Frank J. F. van Veen[9] and Elisa Thébault[10]

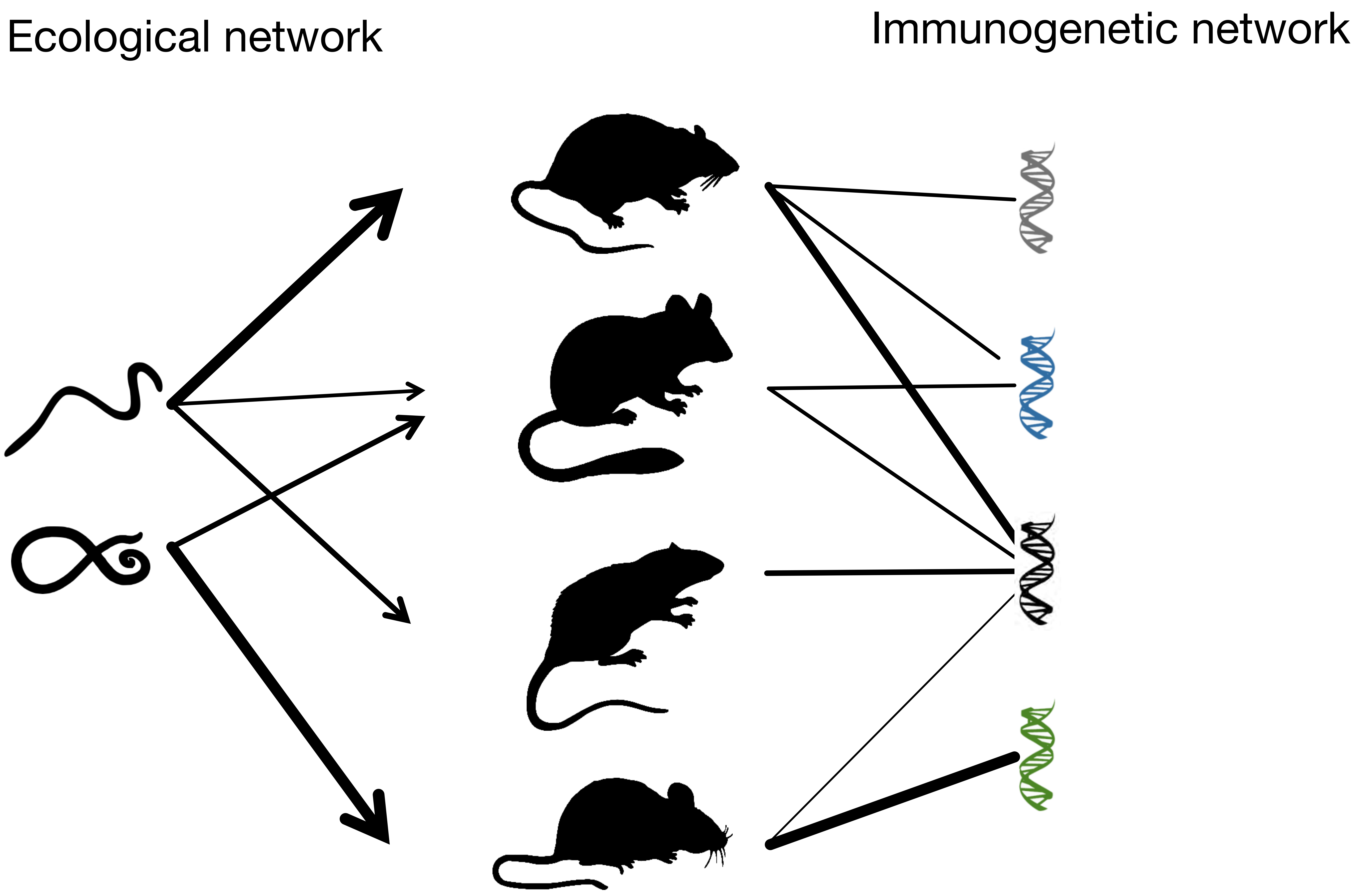


Multiple interaction types and network robustness

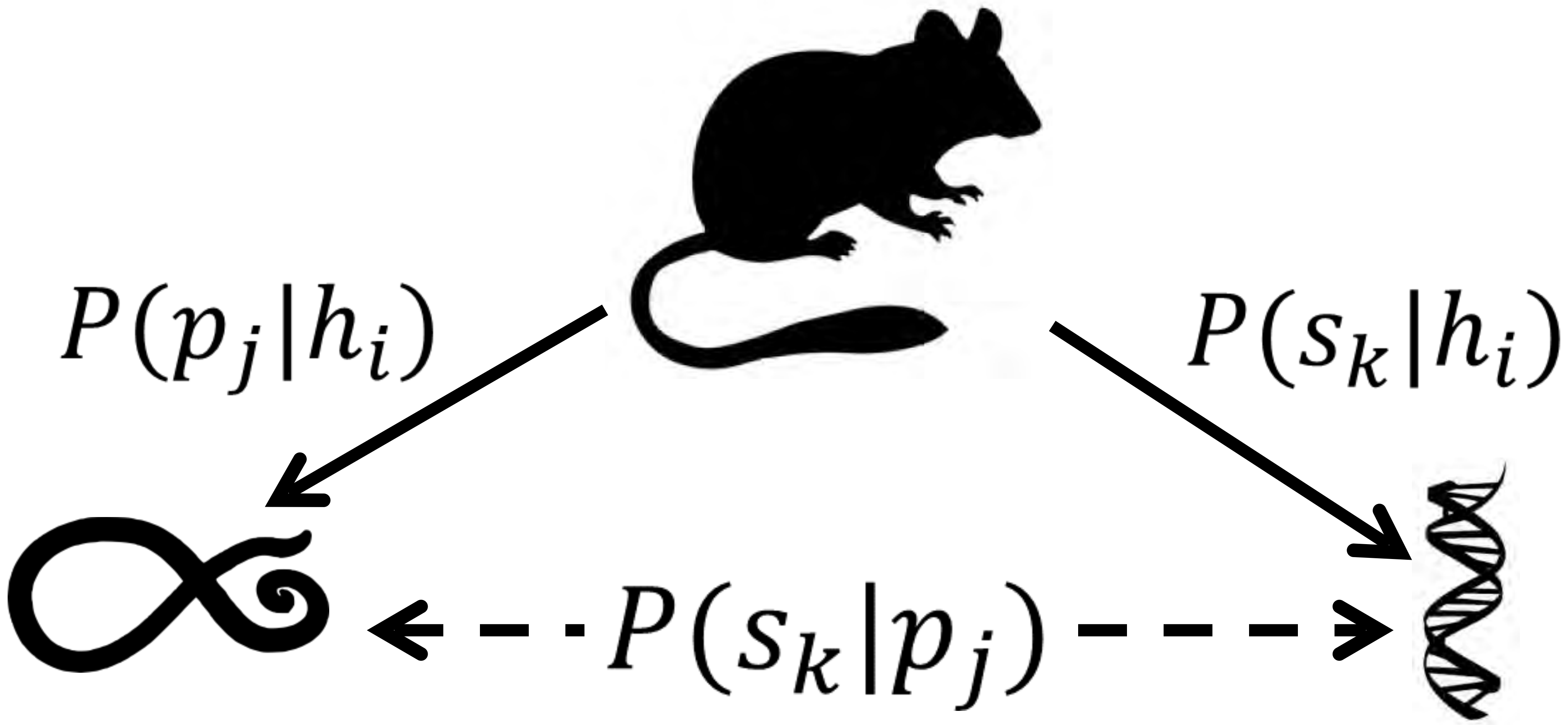




Pocok et al. 2012 Science



Pilosof et al. 2014 Nature Commun.

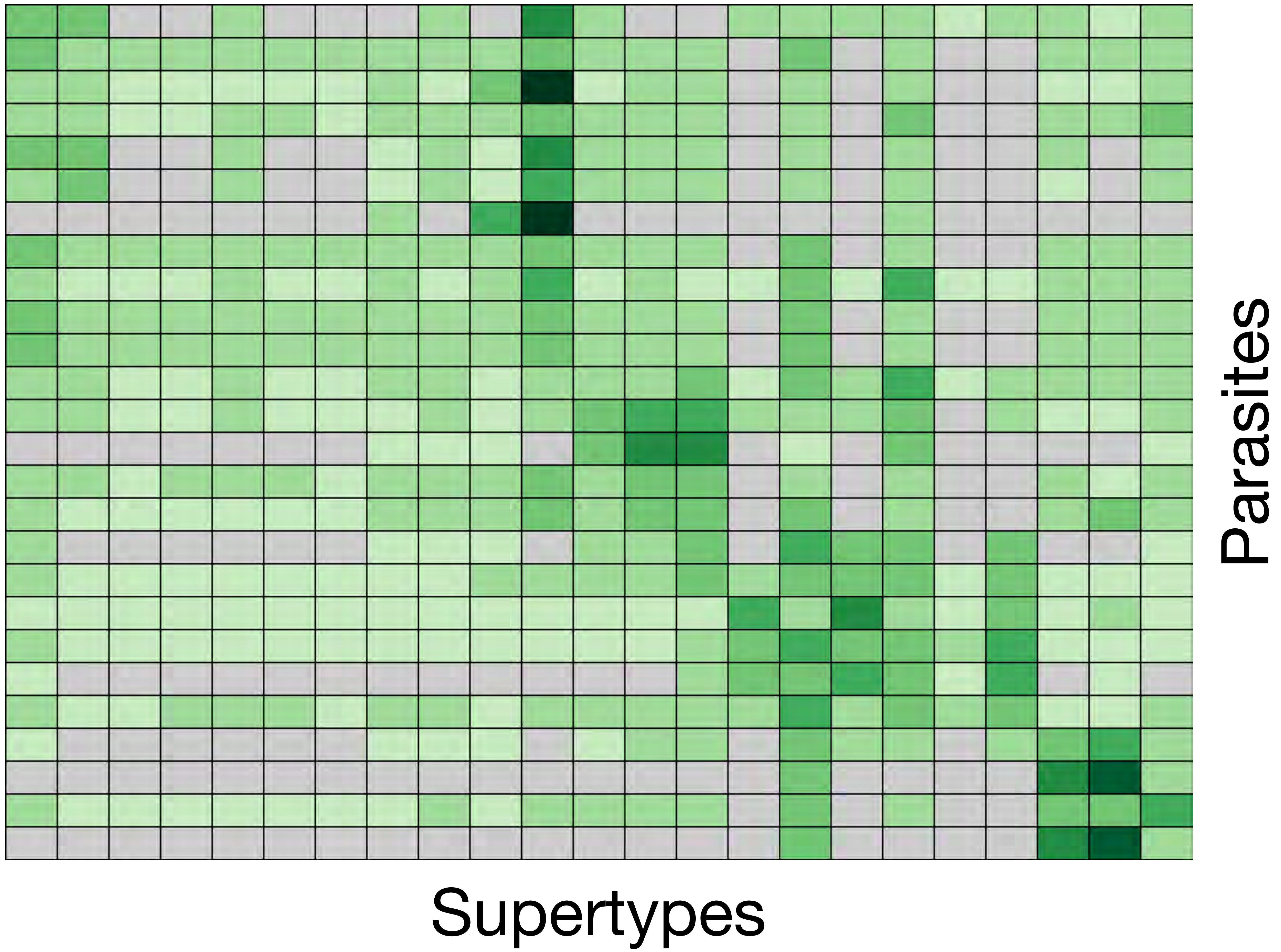


$$P(s = s_k \mid p = p_j) = \tag{1}$$

$$\sum_{i \in H} P(s_k \mid h_i) \cdot P(h_i \mid p_j) = \tag{2}$$

$$\sum_{i \in H} P(s_k \mid h_i) \cdot \frac{P(p_j \mid h_i) \cdot P(h_i)}{P(p_j)} = \tag{3}$$

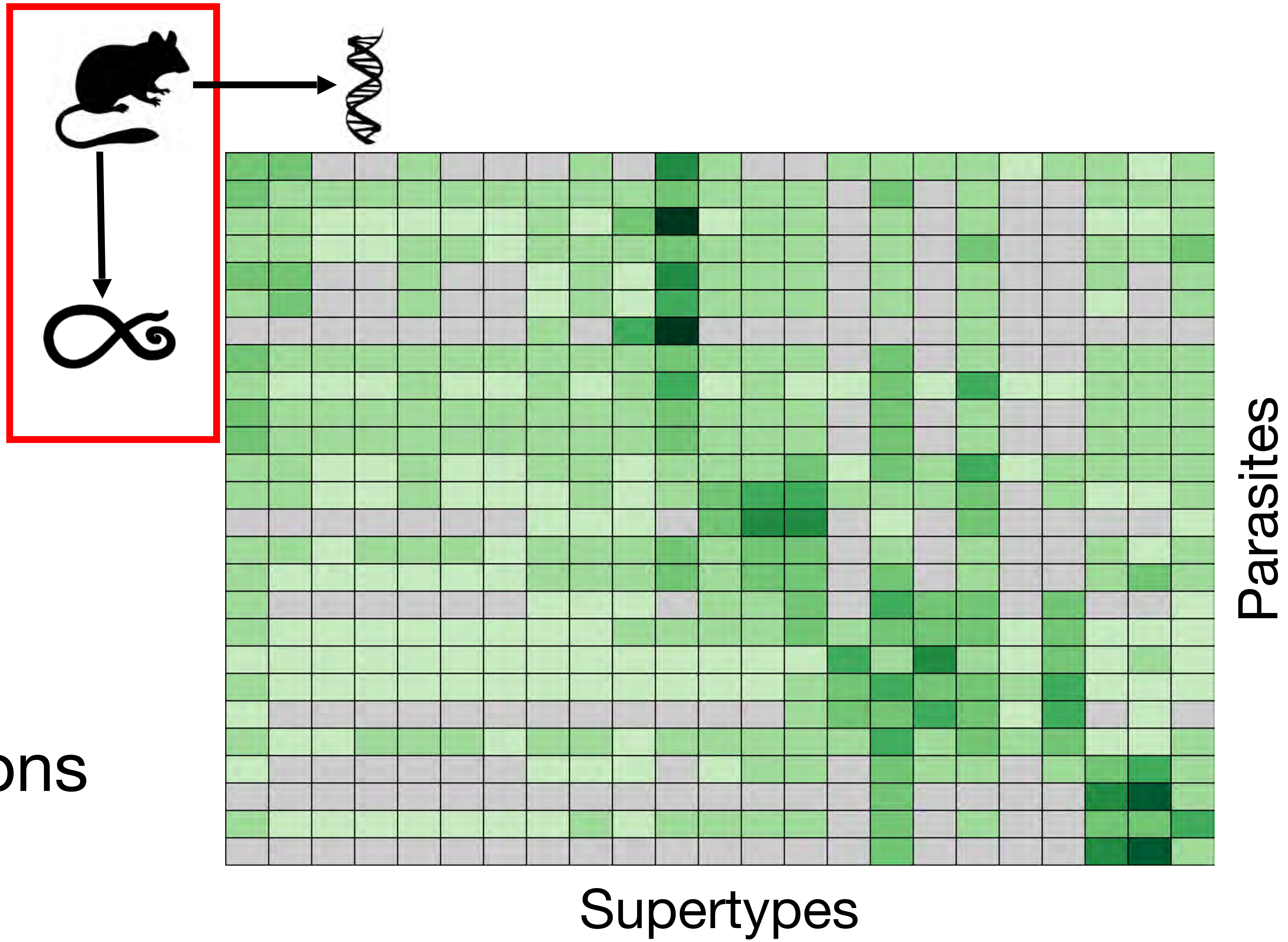
$$\sum_{i \in H} P(s_k \mid h_i) \cdot \frac{P(p_j \mid h_i) \cdot P(h_i)}{\sum_{i \in H} P(h_i) \cdot P(p_j \mid h_i)} \tag{4}$$



Pilosof et al. 2014 Nature Commun.

Does the ecological network determine supertype-parasite associations?

1. Randomize
2. Recalculate



Over 50% of associations are non-random

Pilosof et al. 2014 Nature Commun.


Sergio Navarrete,
Evie Wieters

Kéfi et al. 2015

Trophic interactions

Negative non-trophic interactions

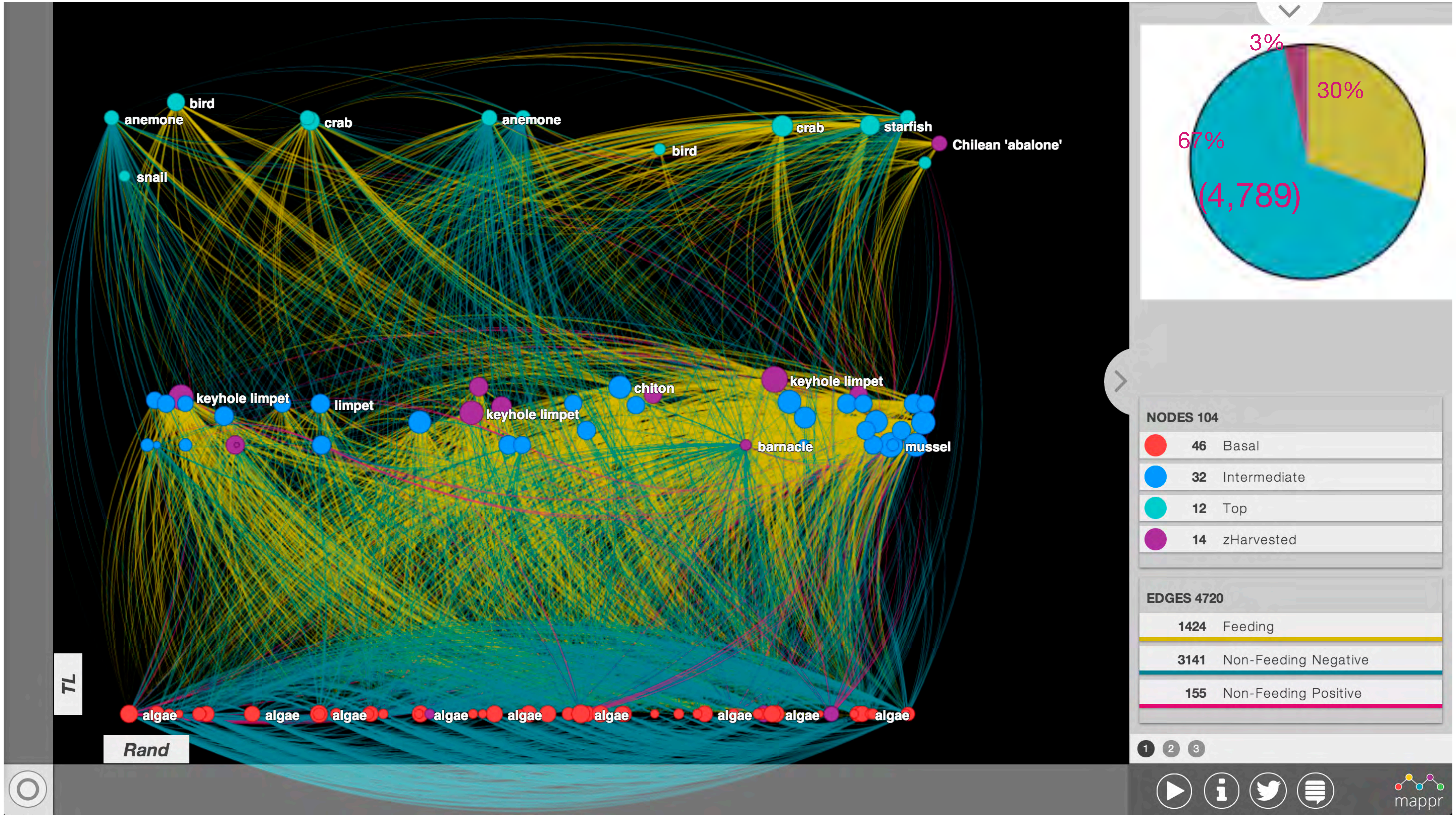
Positive non-trophic interactions

Slide courtesy Sonia Kéfi
Kéfi et al. 2015

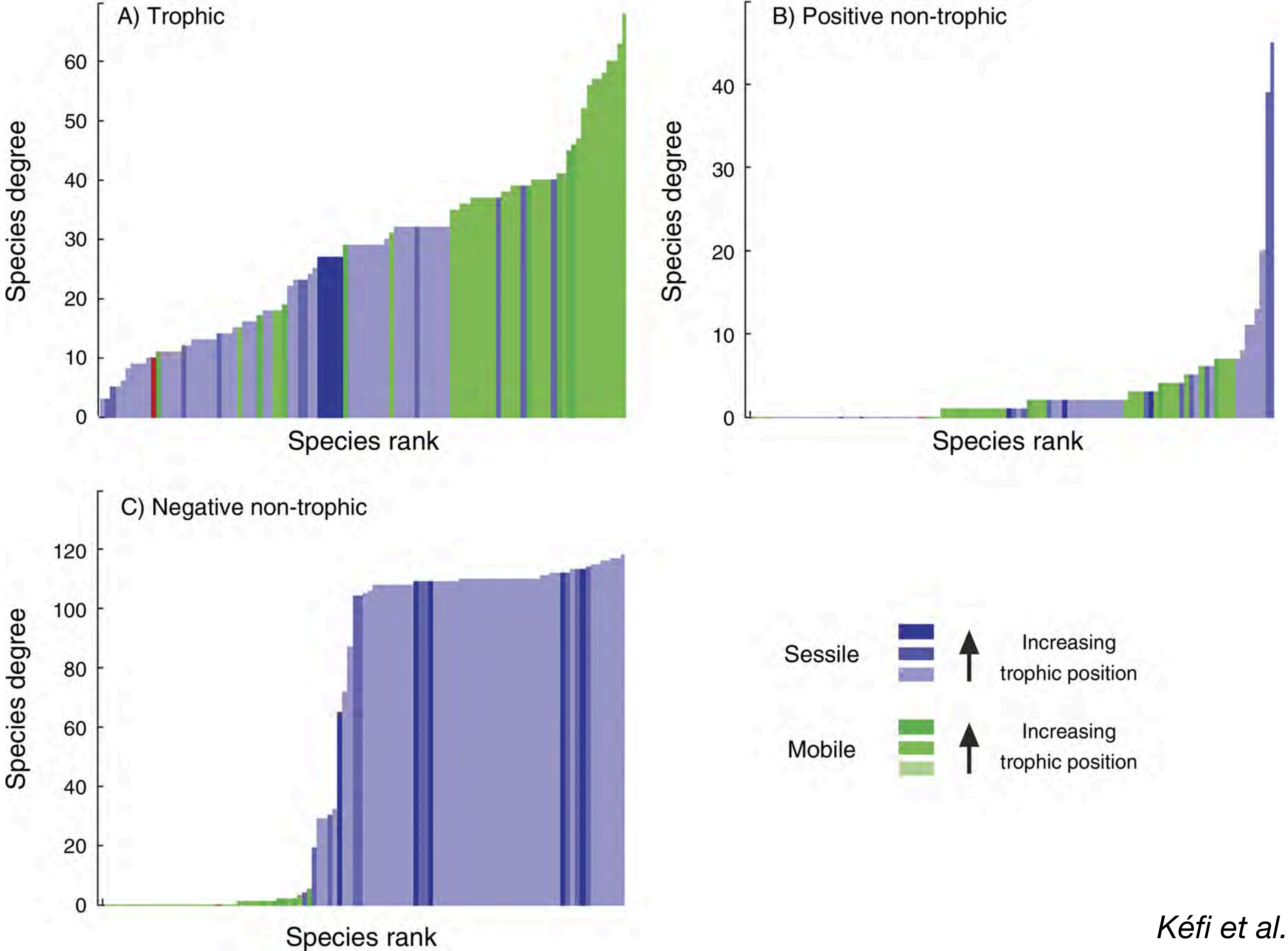
Positive non-trophic interactions

Trophic and non-trophic interactions

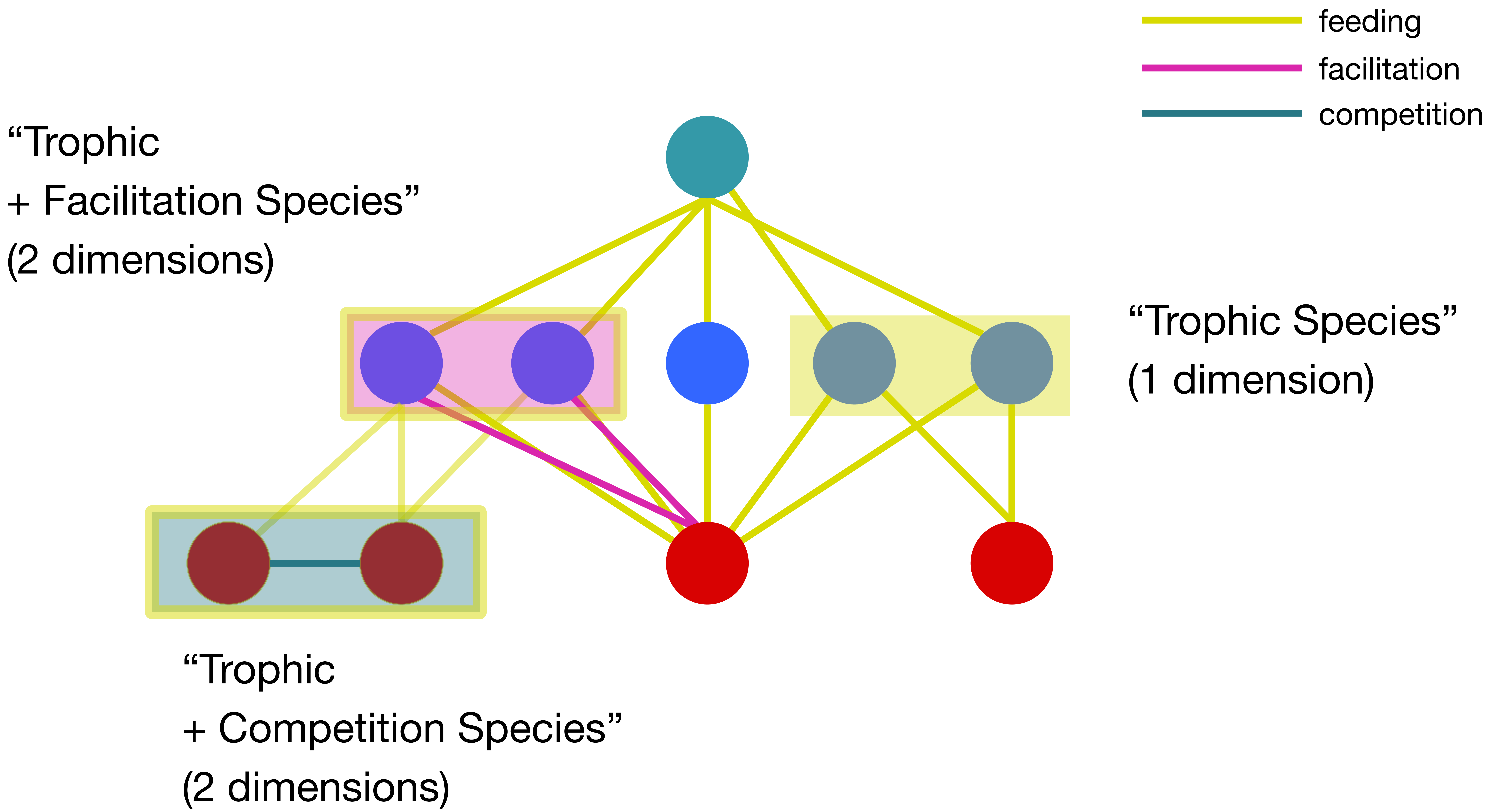
Sonia Kéfi



Kéfi et al. 2015 Ecology

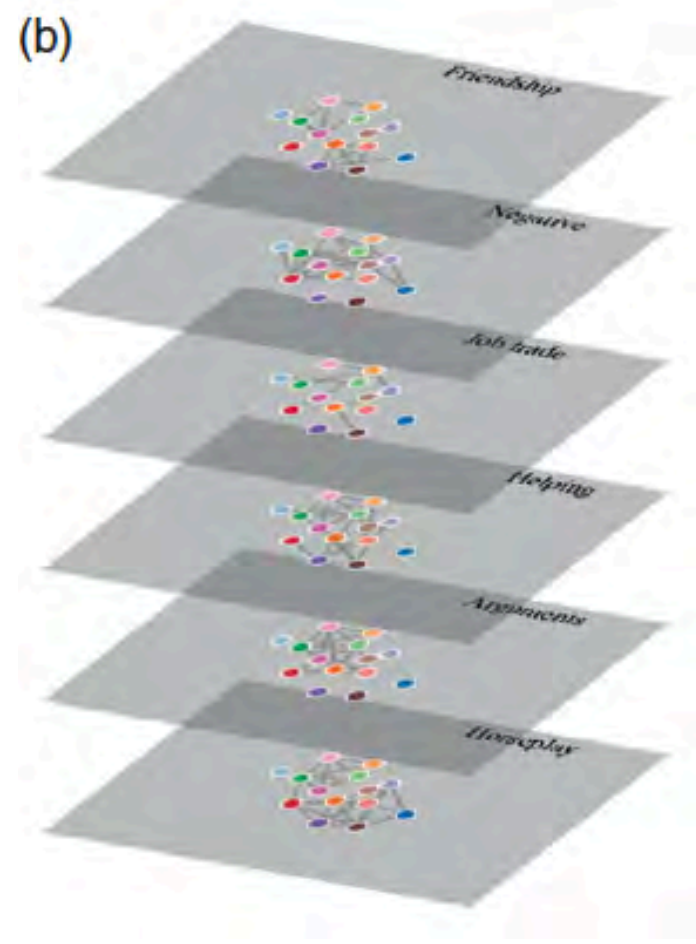


Kéfi et al. 2015 Ecology

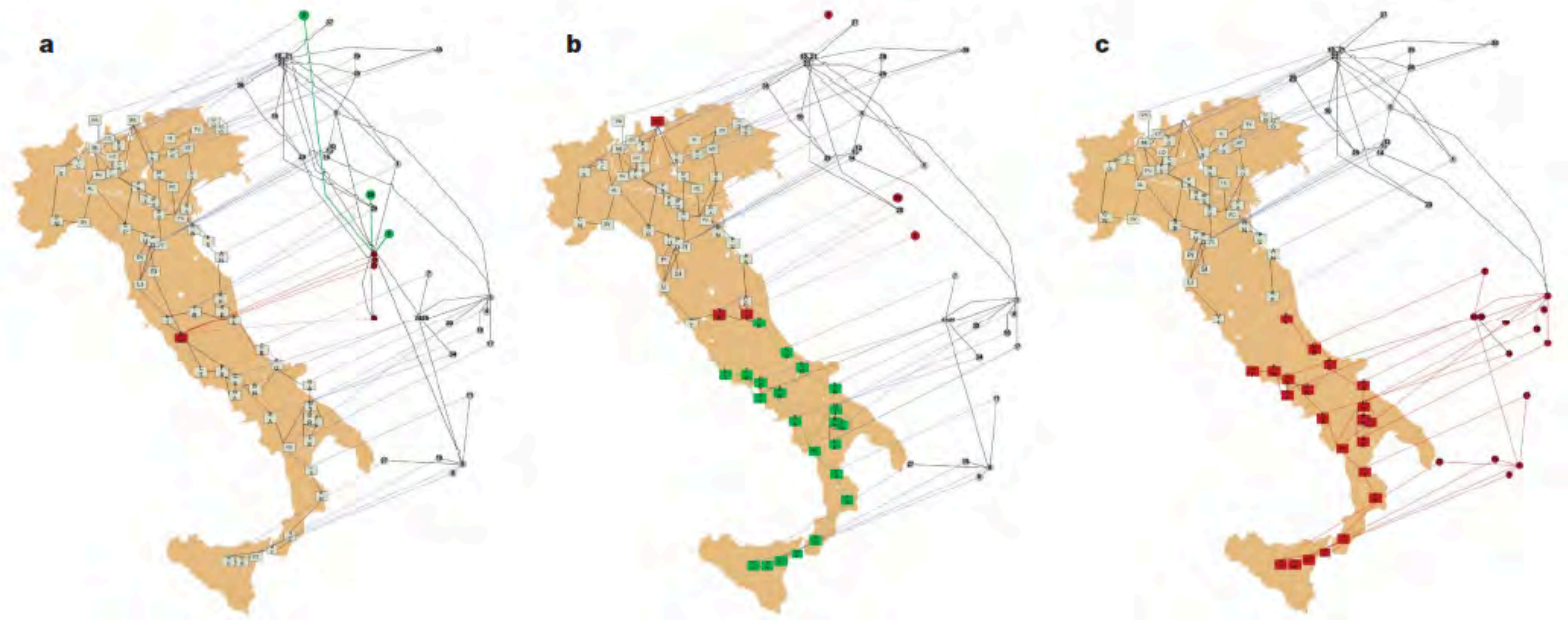


Kéfi et al. 2016 Plos Biol.; Original approach: Baskerville et al. 2011

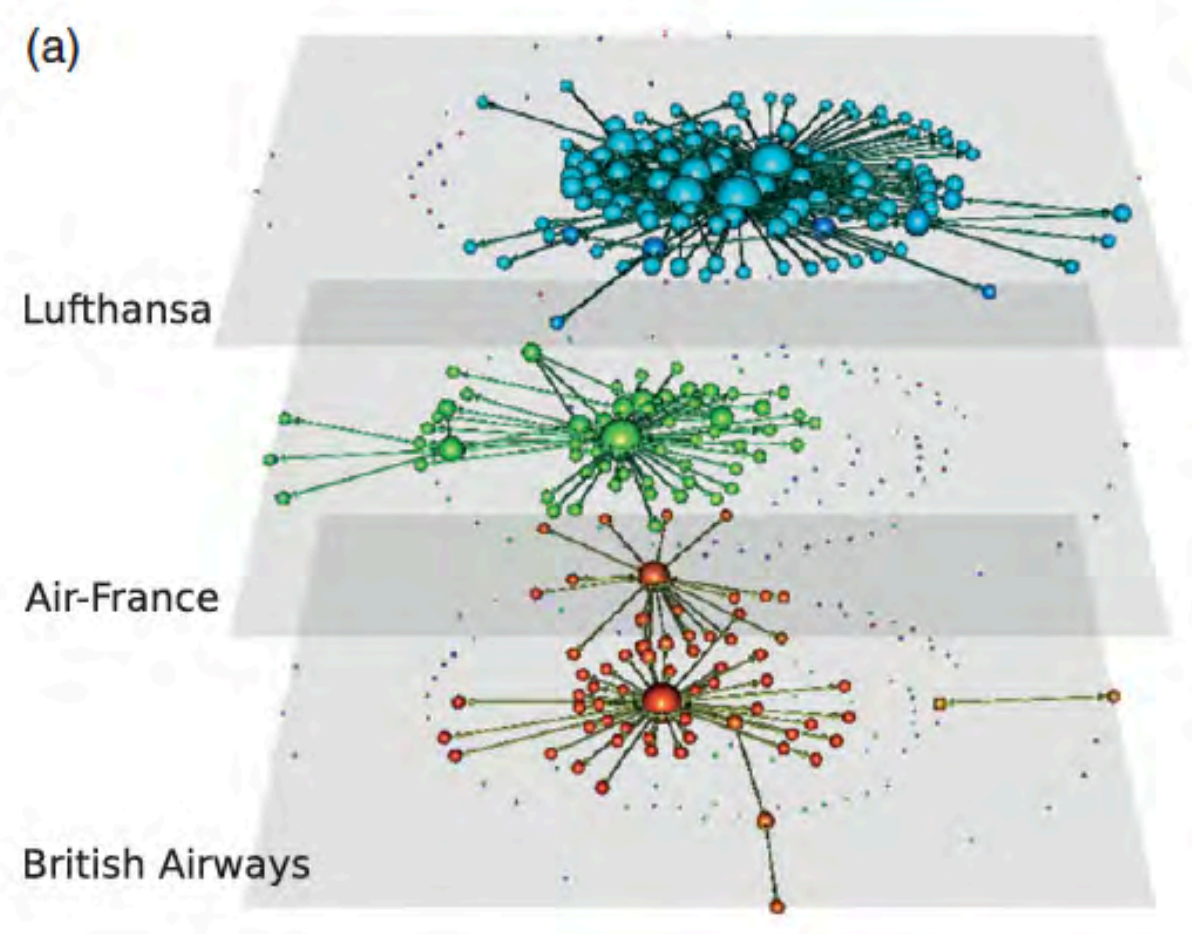
Multilayer networks in the broader network science



Roethlisberger & Dickson (1939)



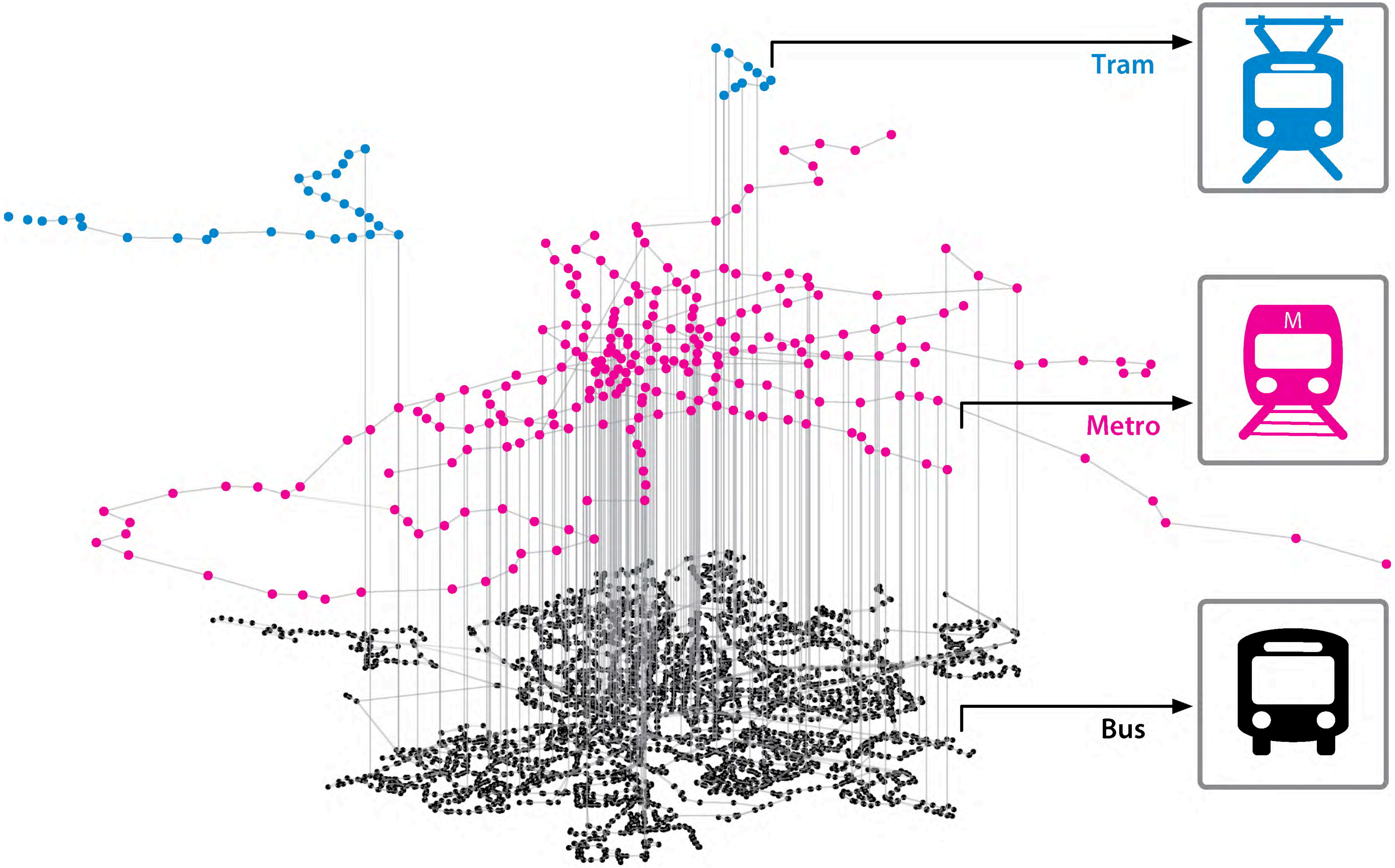
Buldyrev et al. (Nature, 2010)



Cardillo et al. (2013)

Intra-layer links: connected stops of a transport system.

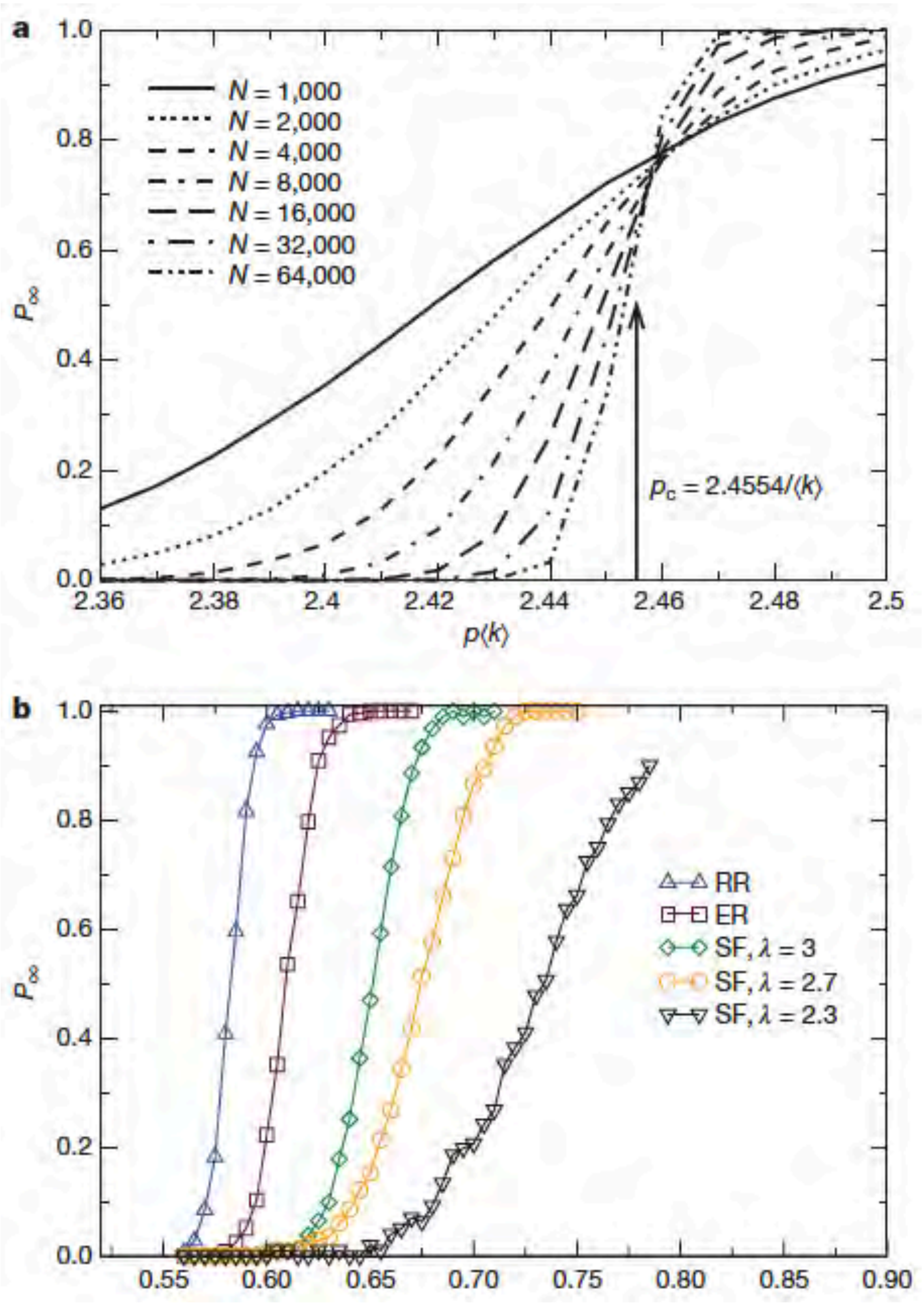
Inter-layer links: connect stops of different transport modes that are within a 150-m radius.

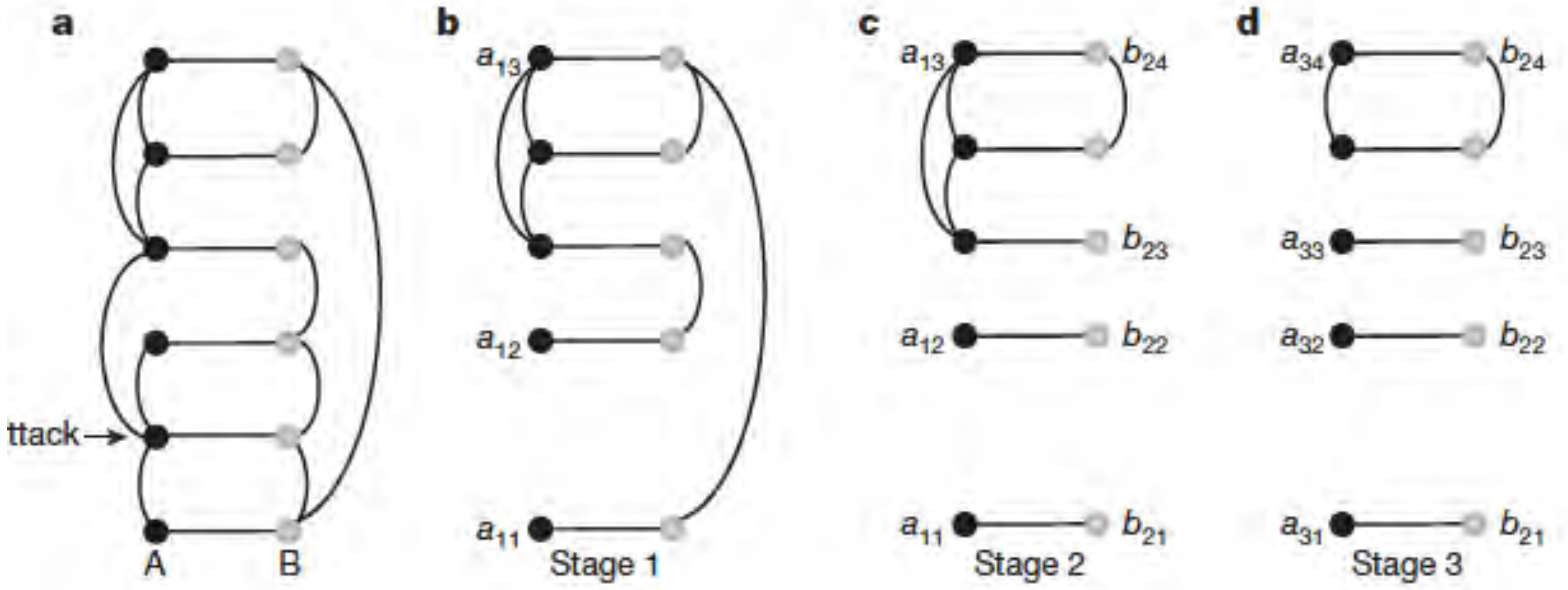


Aleta A, Moreno Y. 2019.
Annu. Rev. Condens. Matter Phys. 10:45–62

Catastrophic cascade of failures in interdependent networks

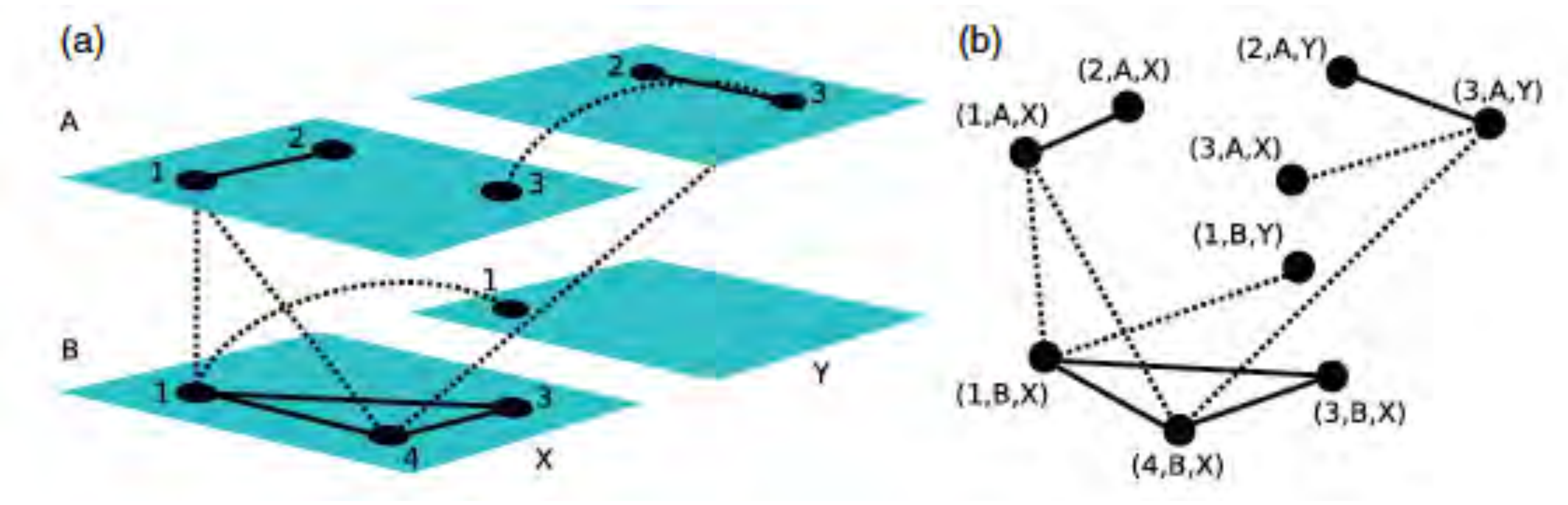
Sergey V. Buldyrev[1,2], Roni Parshani[3], Gerald Paul[2], H. Eugene Stanley[2] & Shlomo Havlin[3]





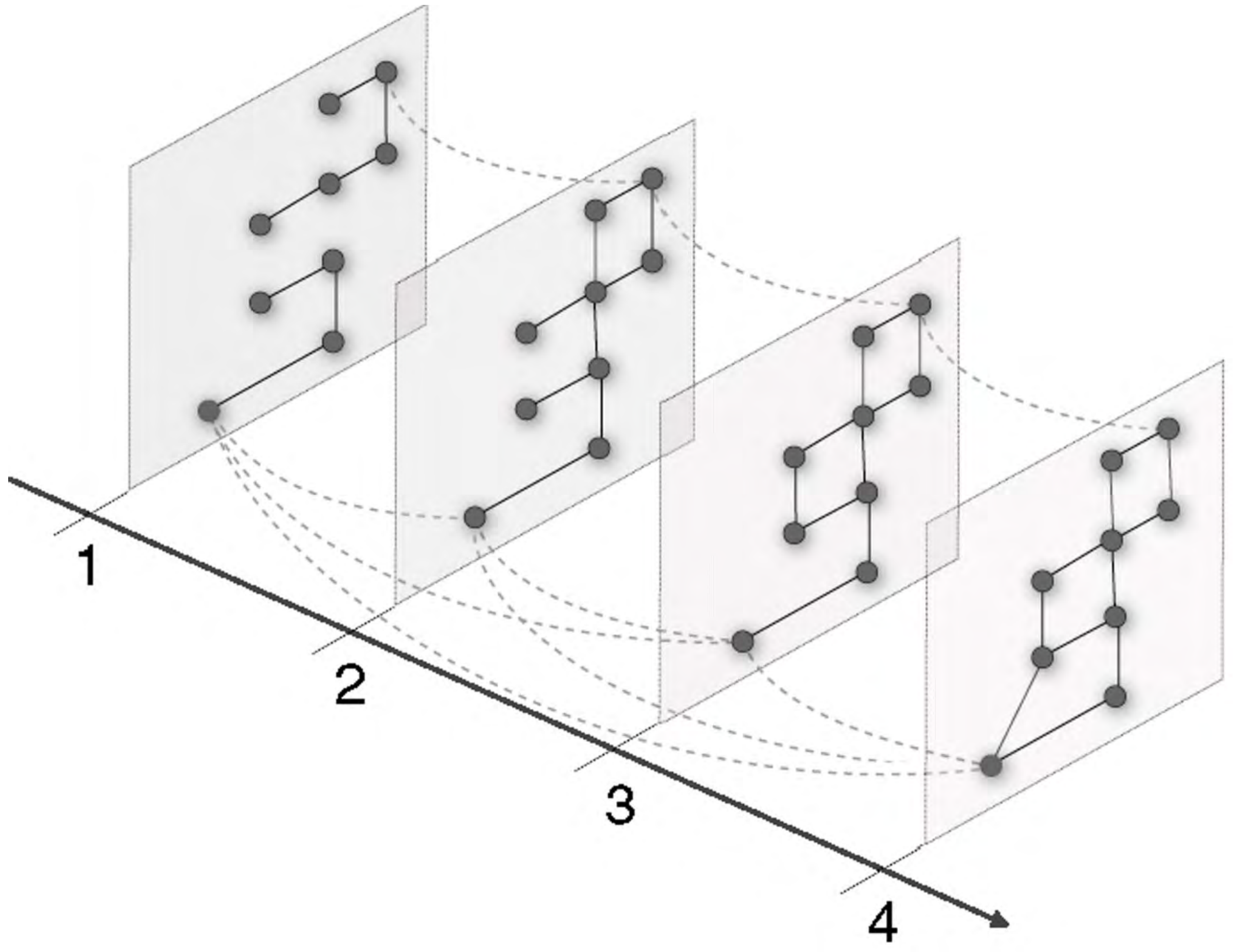
- Couple ER networks.
- Randomly remove nodes.
- Obtain giant mutually connected component.

Buldyrev et al. (Nature, 2010)



General definitions for multilayer networks

Kivelä et al. (2014)



Development of new metrics

So so many…

An example for a generalization to a multilayer metric

$$Q_{\text{multislice}} = \frac{1}{2\mu}\sum_{ijsr}\left[\left(A_{ijs} - \gamma_s \frac{k_{is}k_{js}}{2m_s}\right)\delta_{sr} + \delta_{ij}\omega_{jsr}\right]\delta(g_{is}, g_{jr})$$

within-layer

between-layer

Mucha et al. 2010 Science

nature ecology & evolution

PERSPECTIVE

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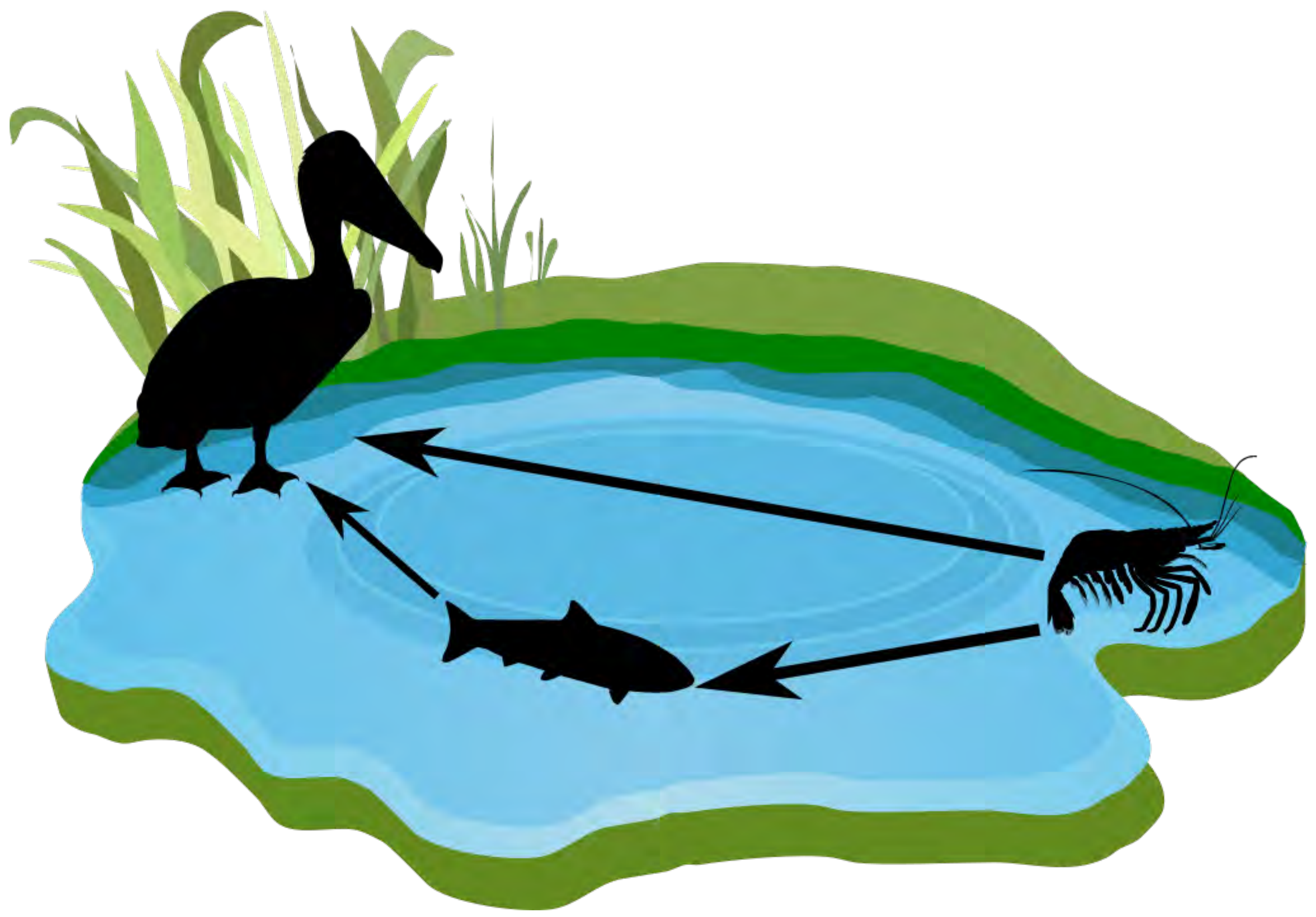
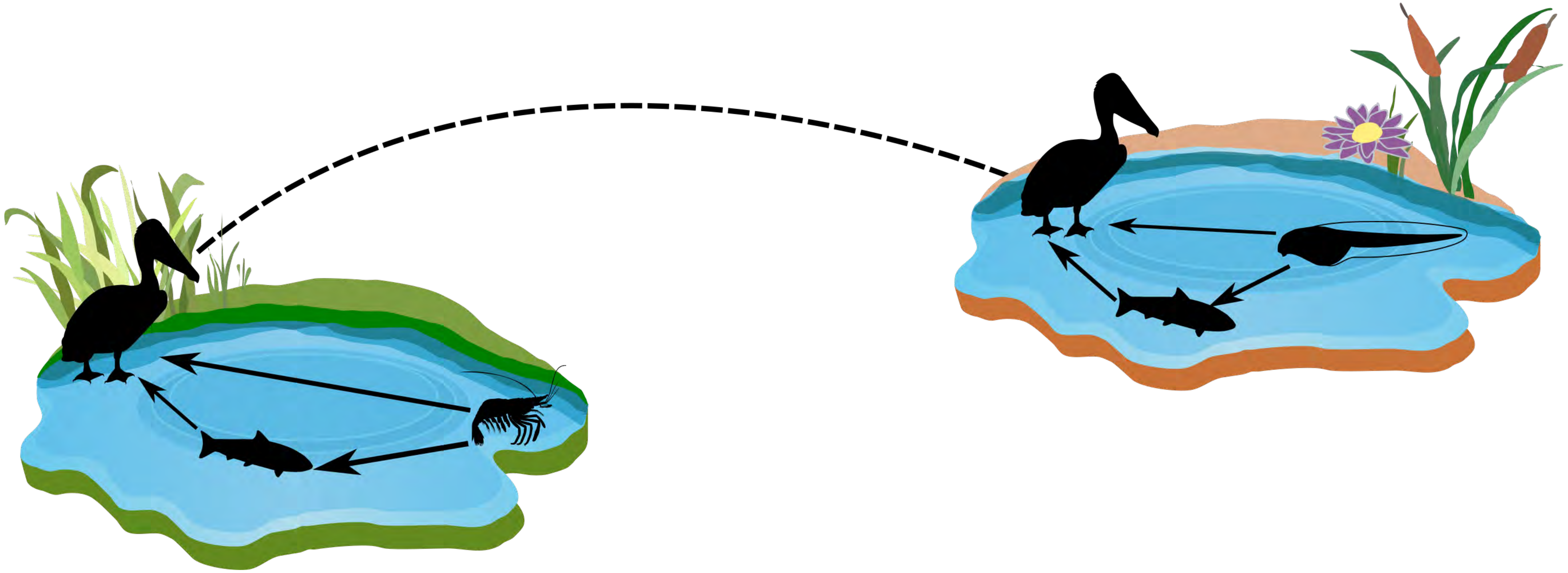
The multilayer nature of ecological networks

Shai Pilosof[1*], Mason A. Porter[2,3,4], Mercedes Pascual[1,5] and Sonia Kéfi[6]

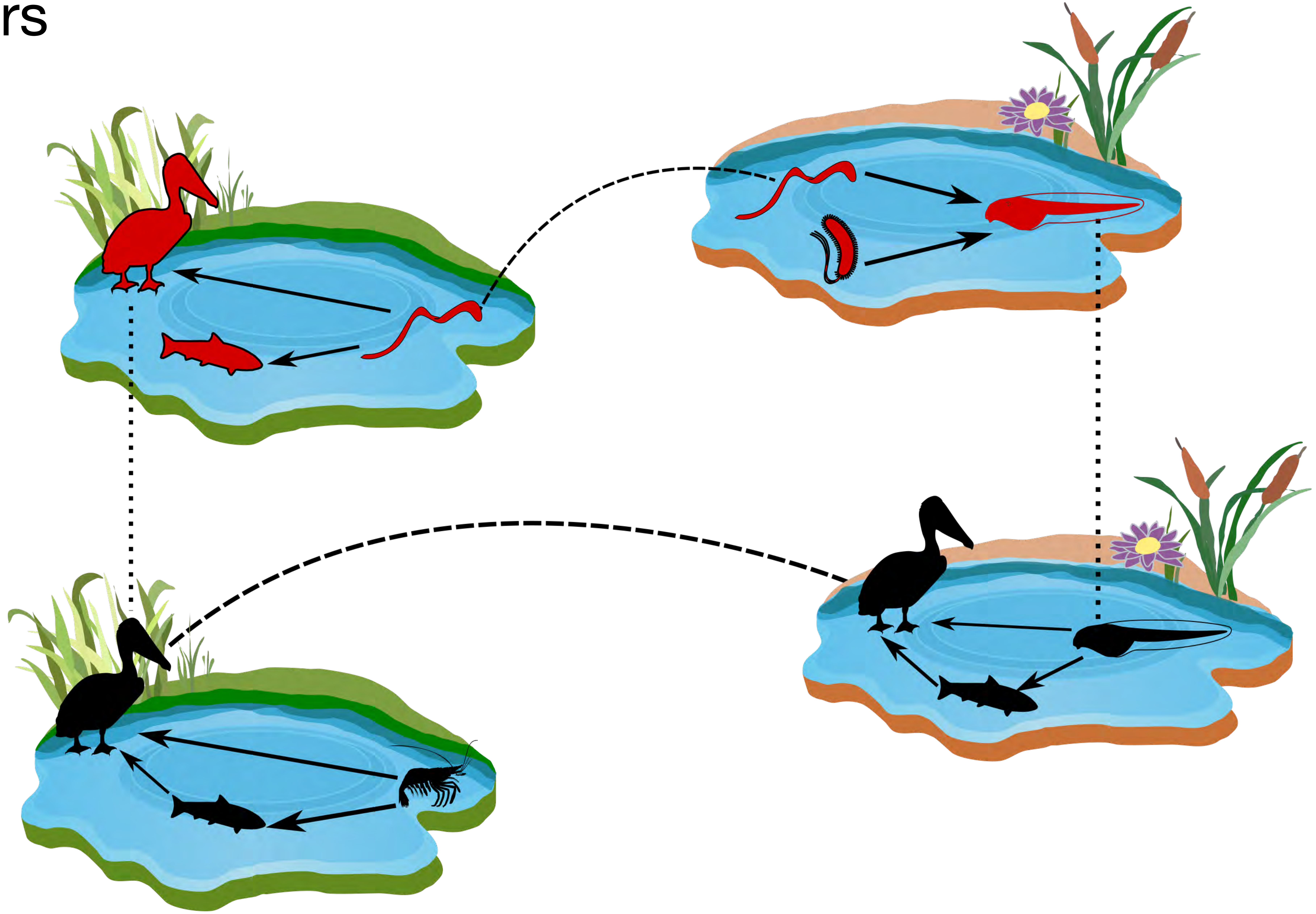




Pilosof et al. 2017 Nature Ecol. Evol.



- Dimensions, layers
- State nodes
- Intra-layer edges
- Inter-layer edges



Pilosof et al. 2017 Nature Ecol. Evol.

$$\mathbf{L} = \{L_a\}_{a=1}^{d}$$

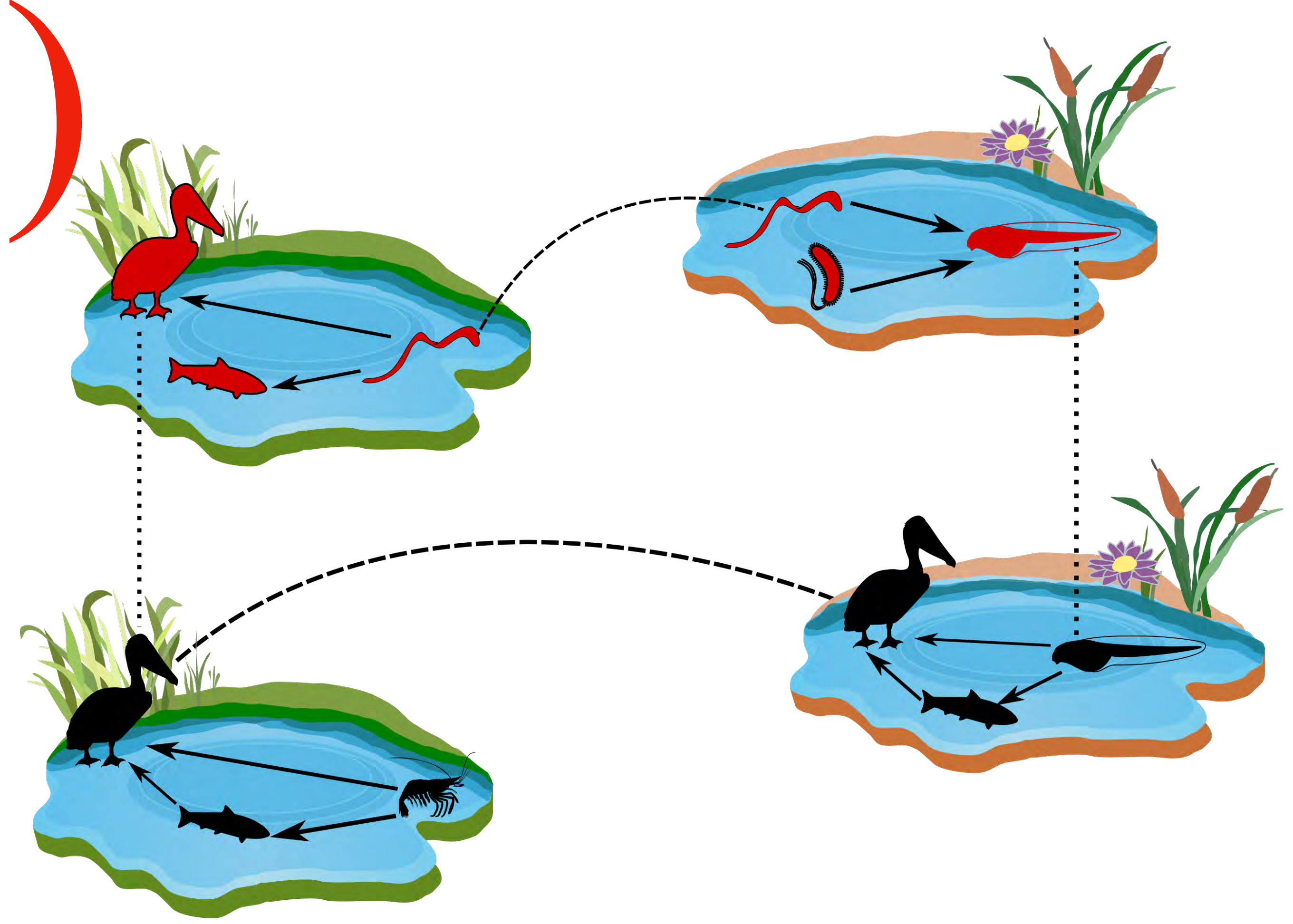
$$V_M \subseteq V \times L_1 \times L_2 \cdots \times L_d$$

$$E_M \subseteq V_M \times V_M$$

$$M = (V_M, E_M, V, \mathbf{L})$$

(u,α)

(u,β)



Kivelä et al. 2014 J. Complex. Networks

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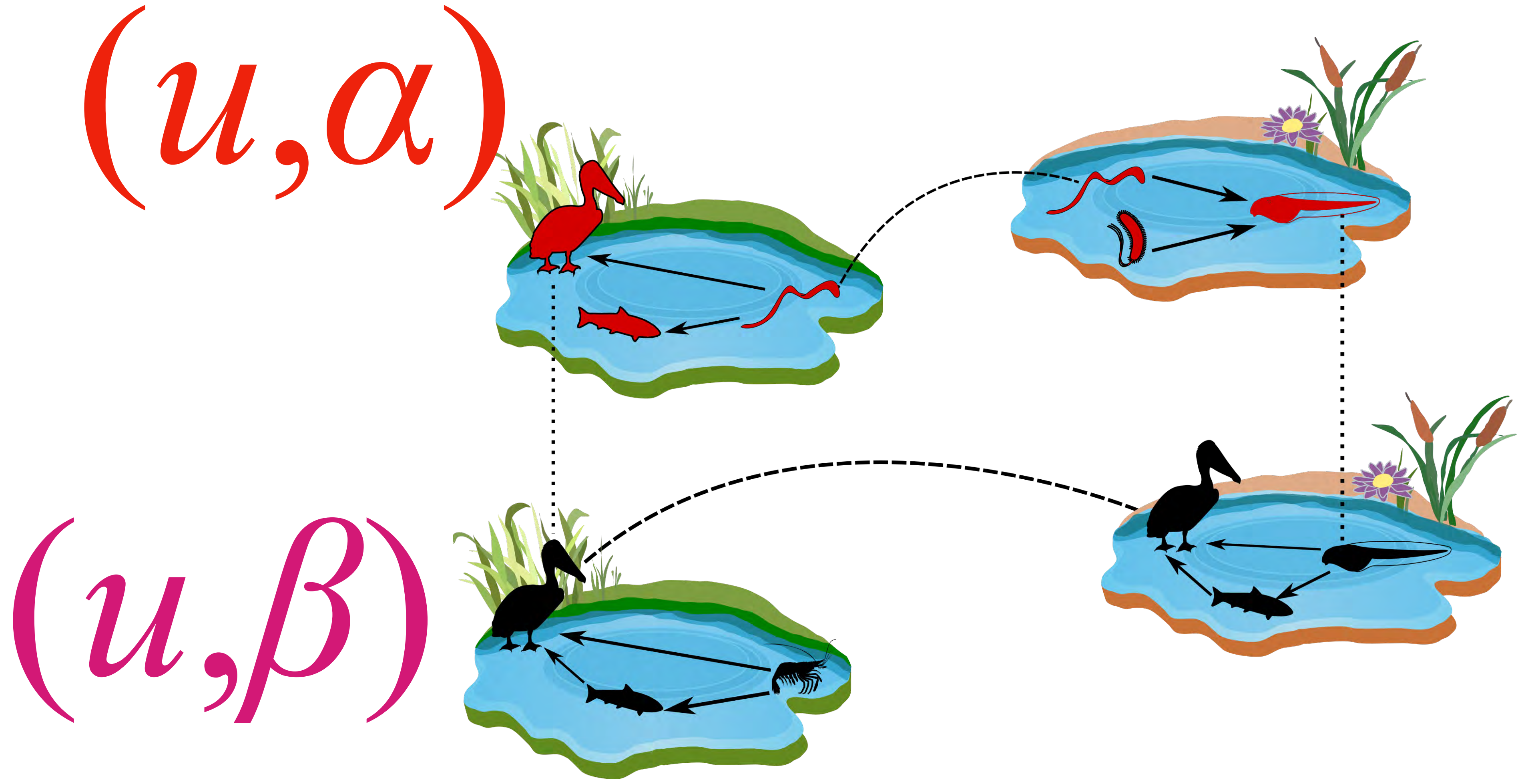

$$E_A = \{((u, \alpha), (v, \beta)) \in E_M | \alpha = \beta\}$$

$$E_C = E_M \setminus E_A$$

Kivelä et al. 2014 J. Complex. Networks

$$\mathbf{L} = \{L_a\}_{a=1}^{d}$$

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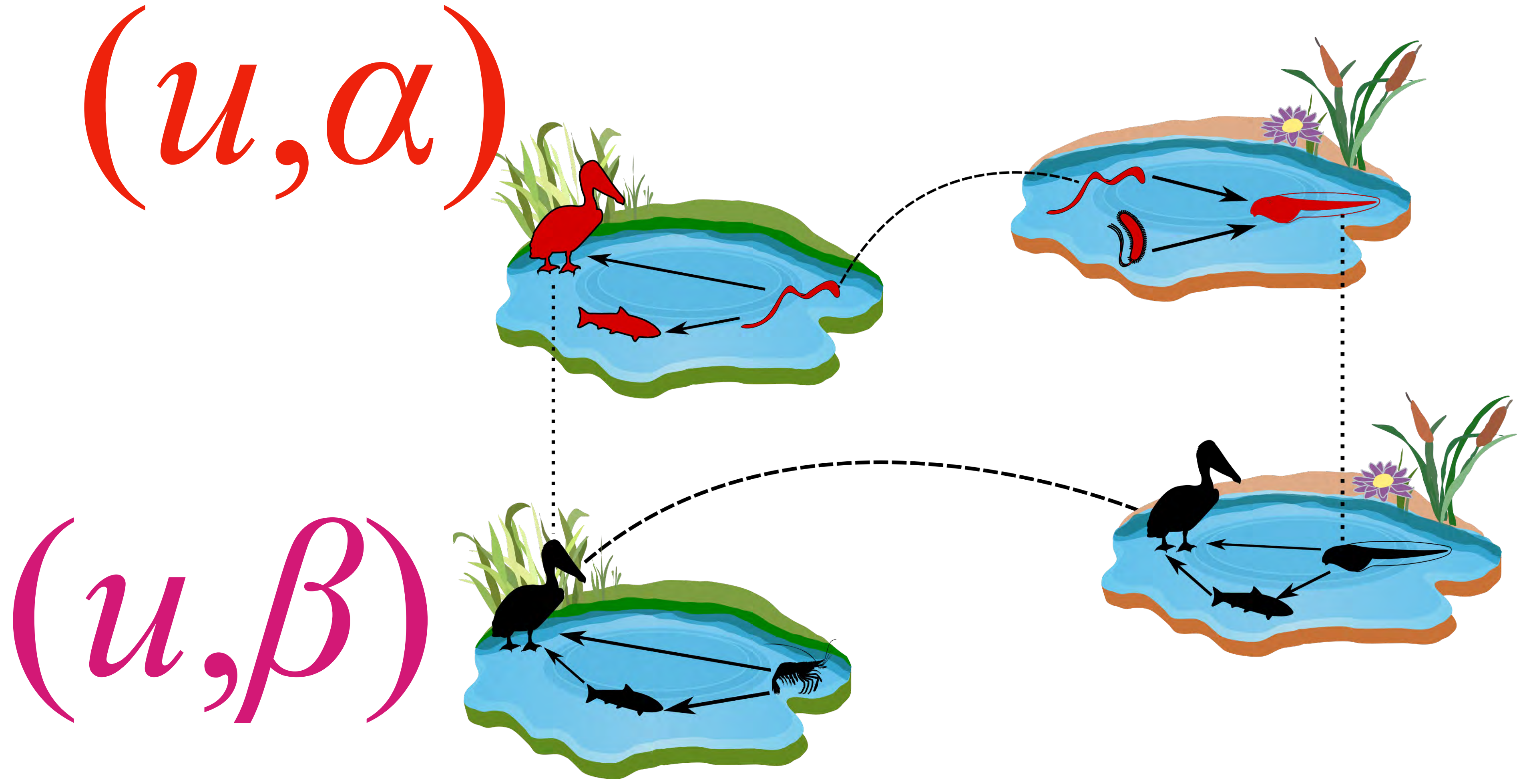


$$E_A = \{((u,\alpha),(v,\beta)) \in E_M | \alpha = \beta\}$$

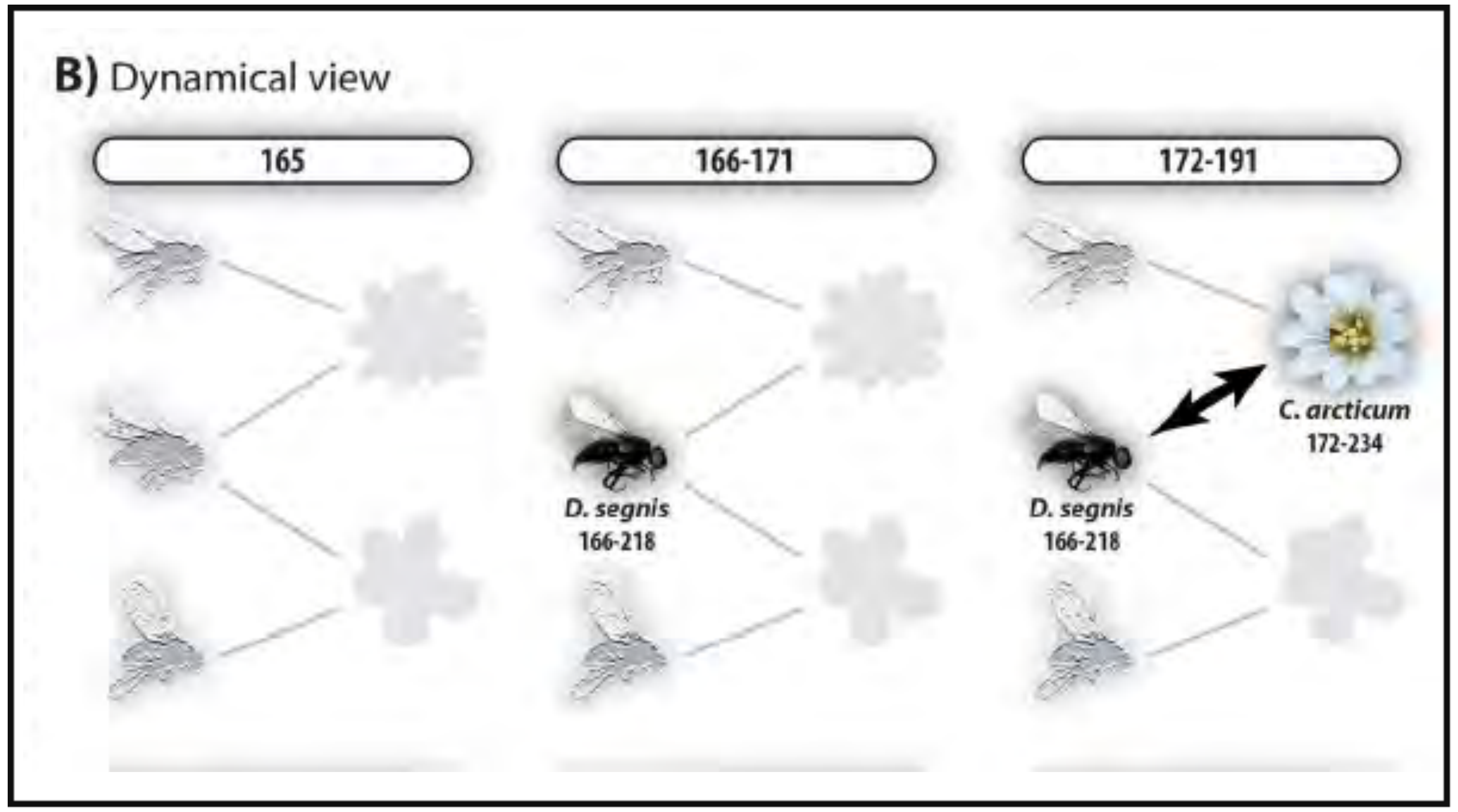
$$E_C = E_M \setminus E_A$$

$$E_T = \{((u,t),(v,t+\tau)) \in E_C\}$$

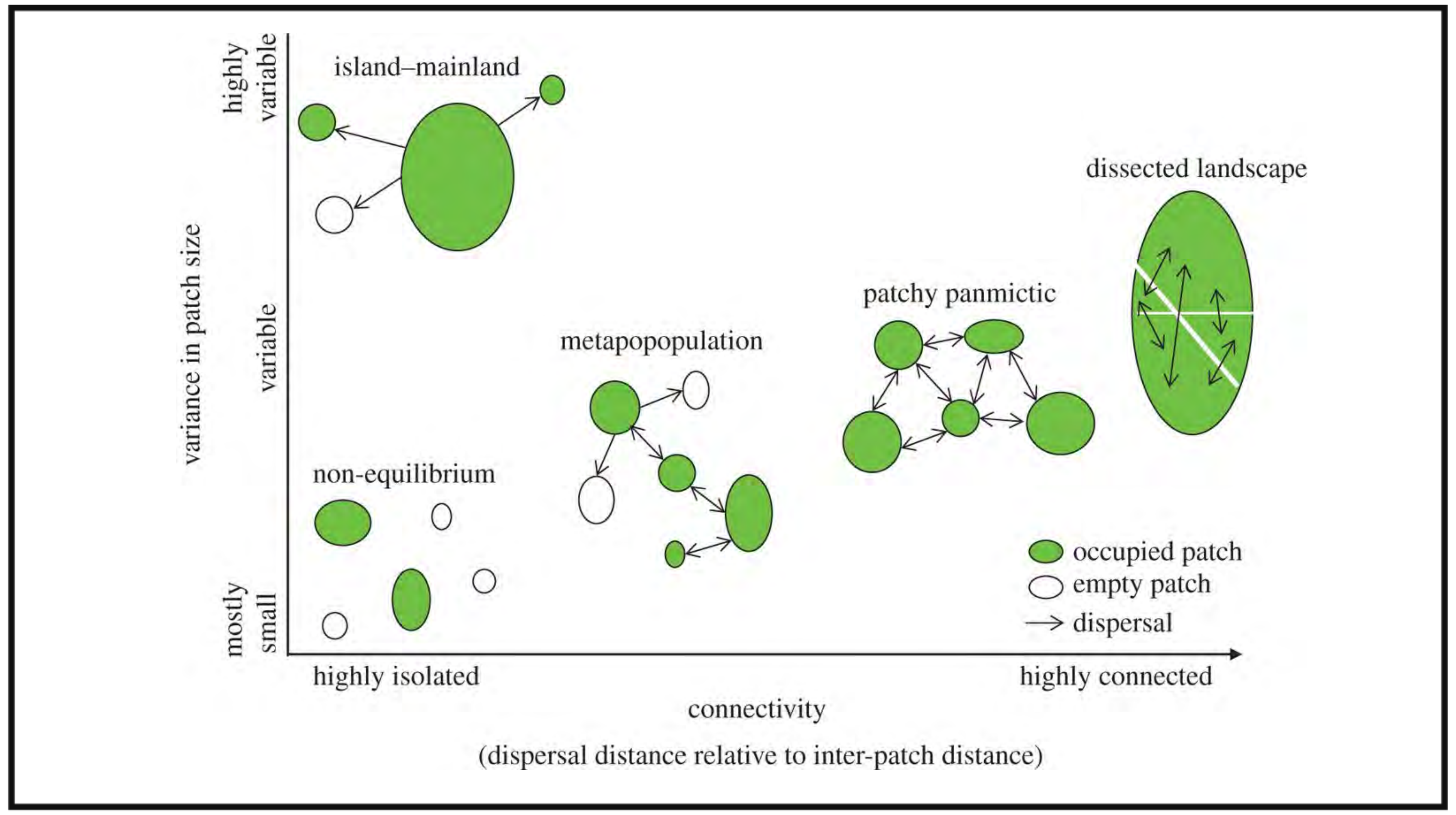
Temporal network

Kivelä et al. 2014 J. Complex. Networks

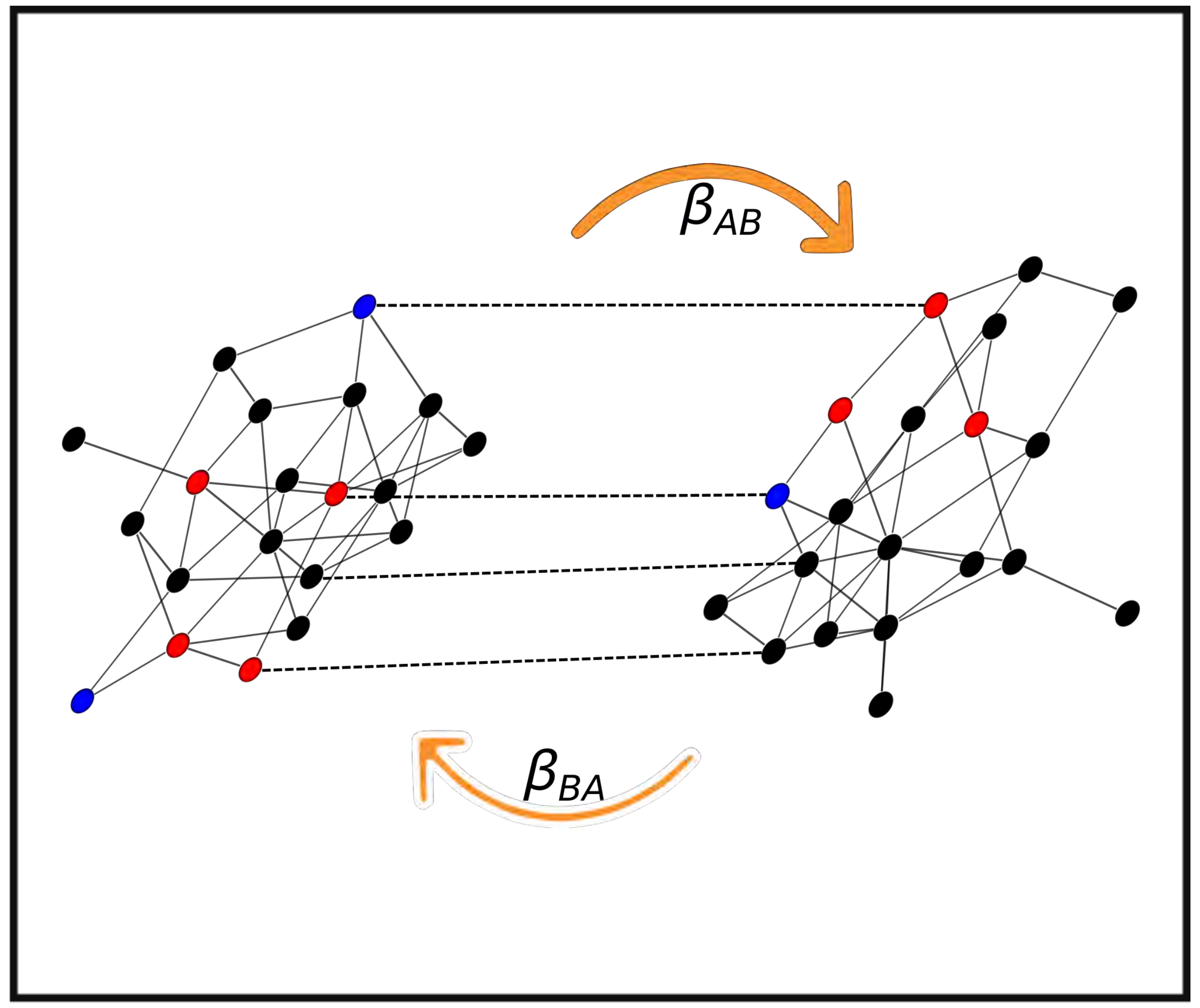
Categories of Ecological Multilayer networks



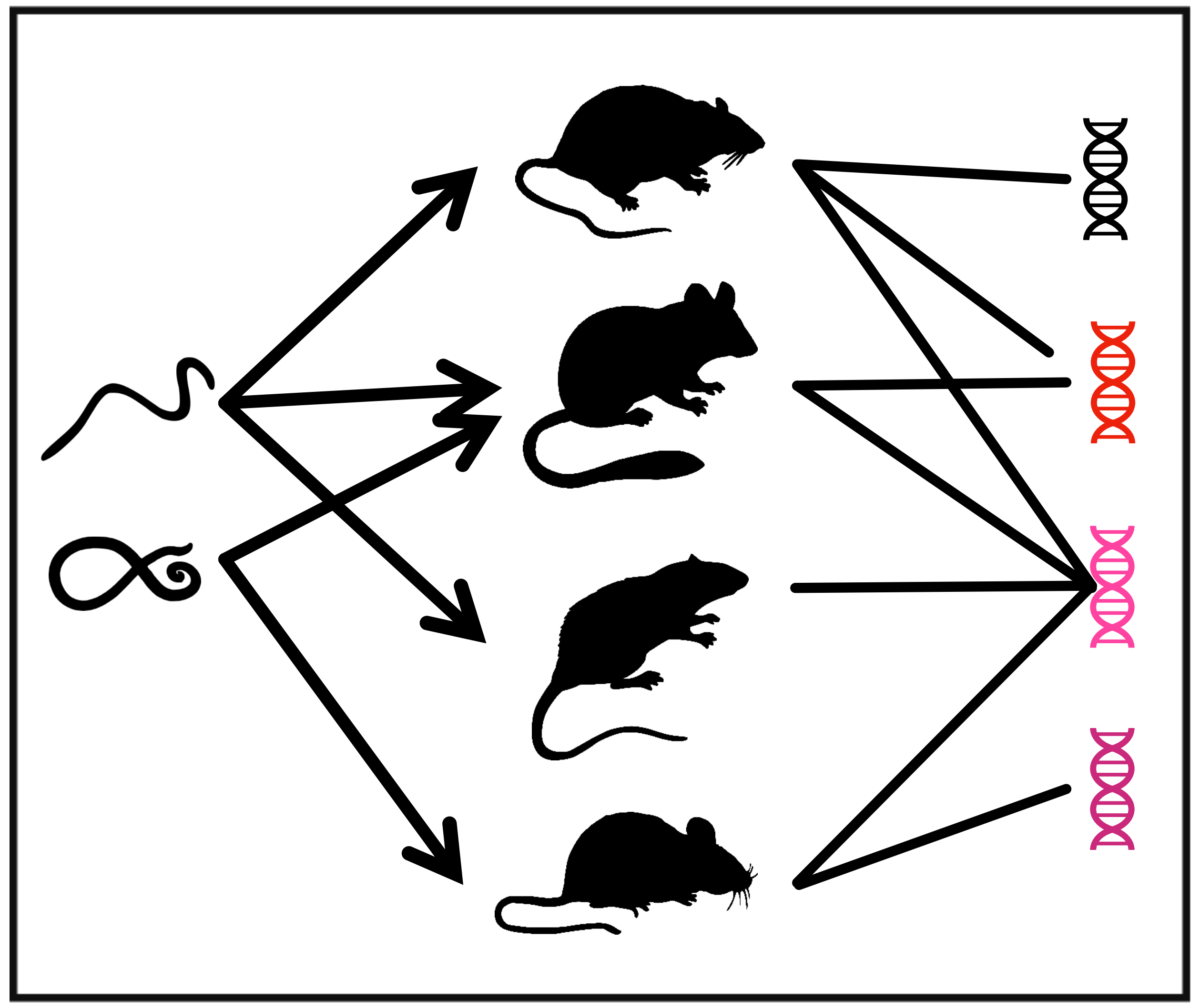
Rasmussen et al. 2013



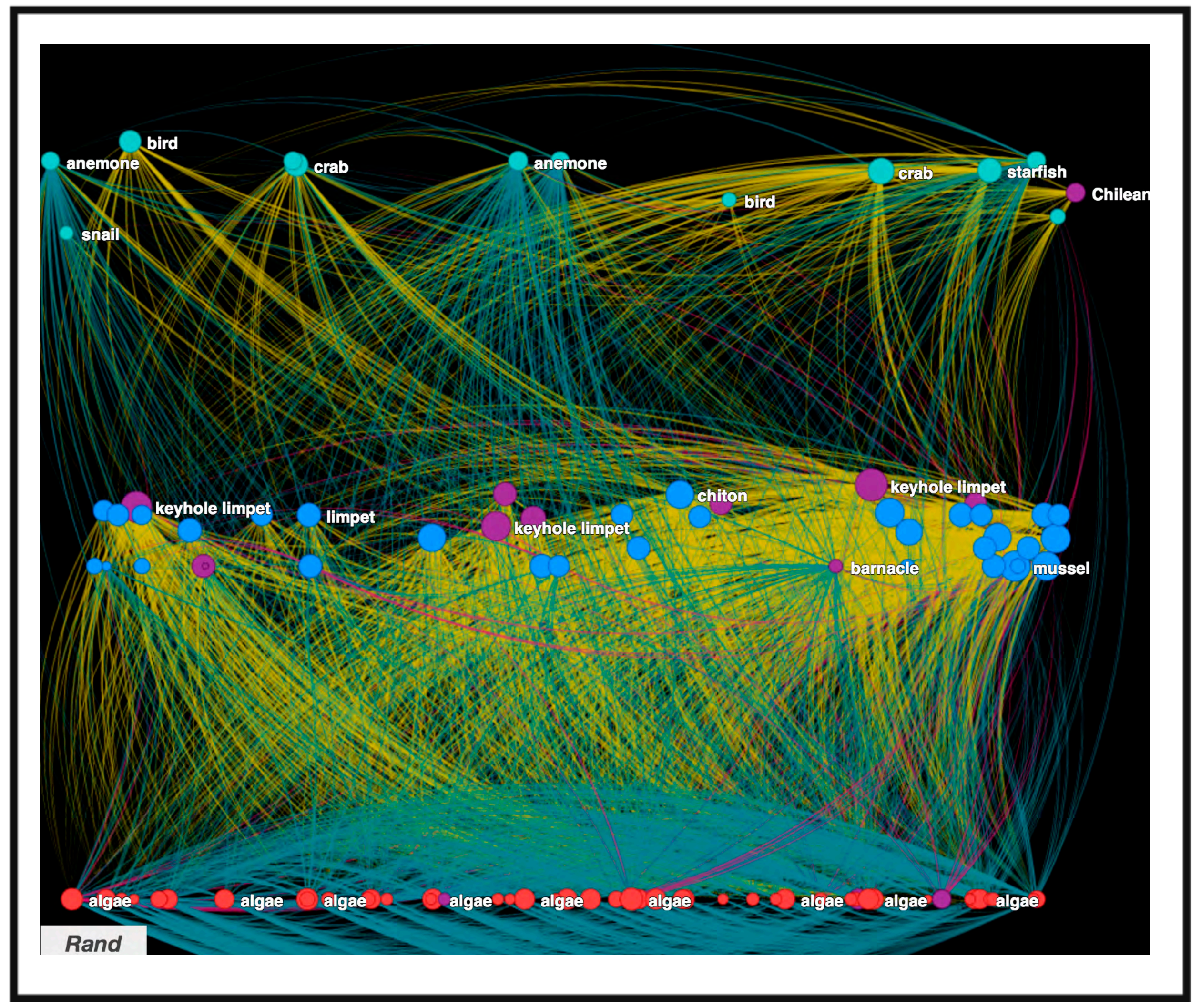
Cheptou et al., 2014



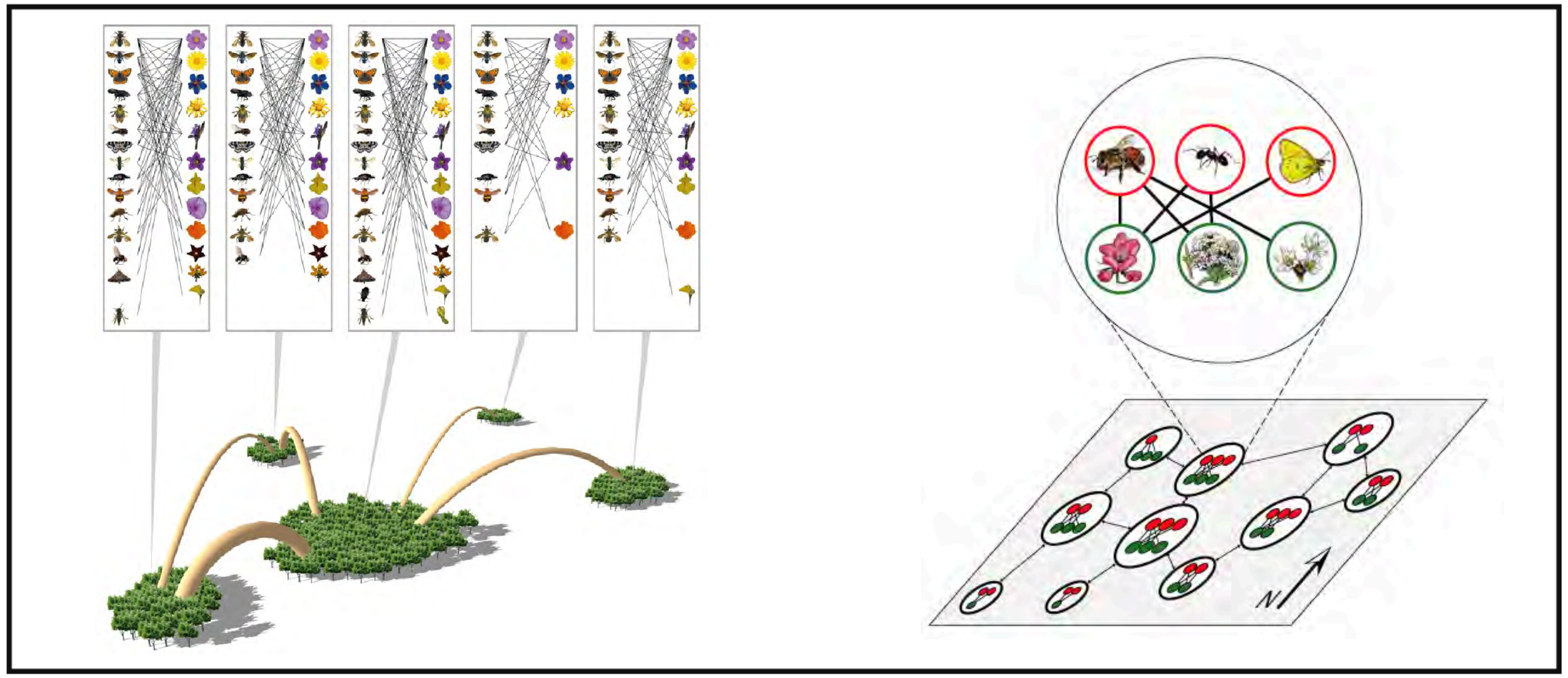
Pilosof et al., 2017



Pilosof et al. 2014

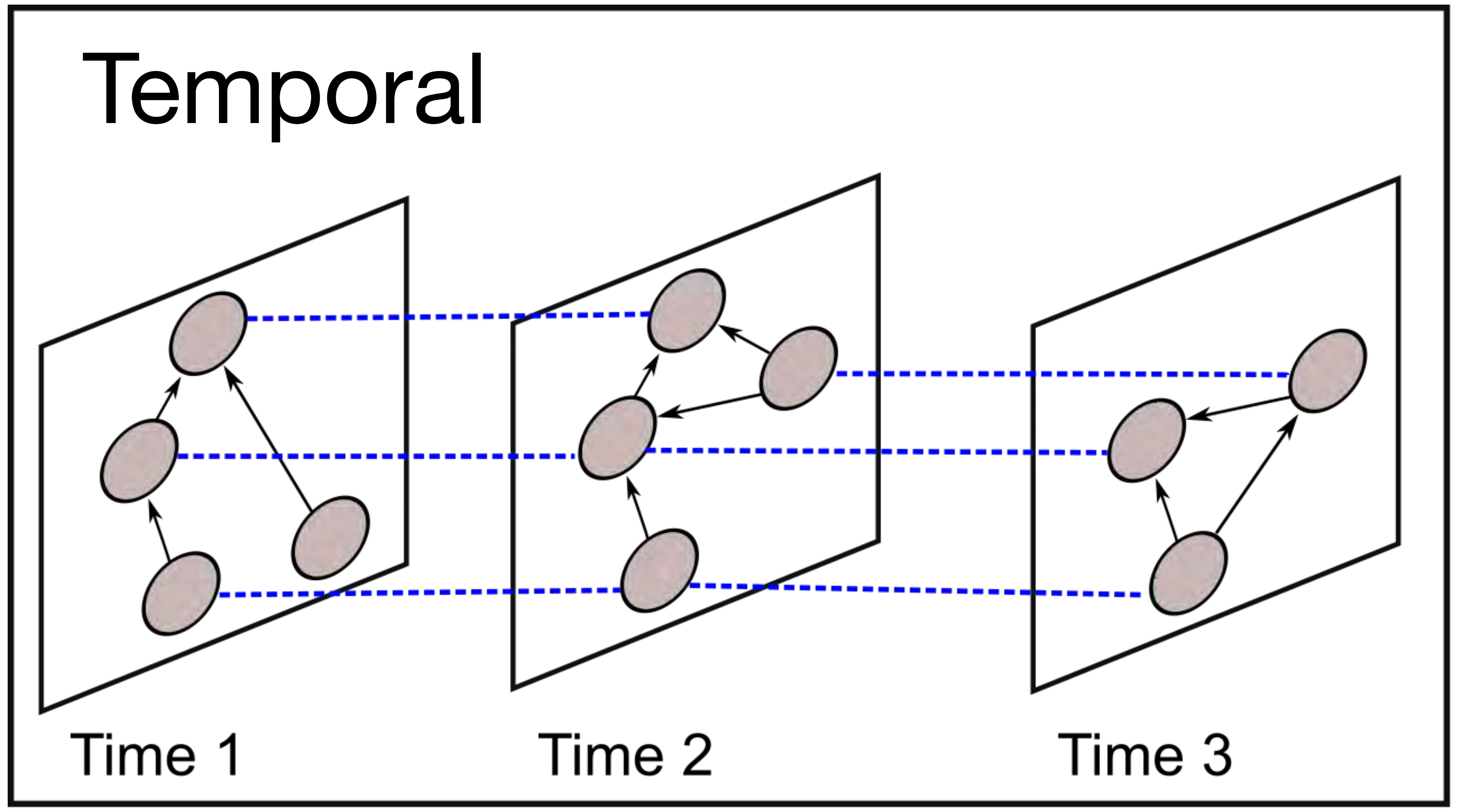


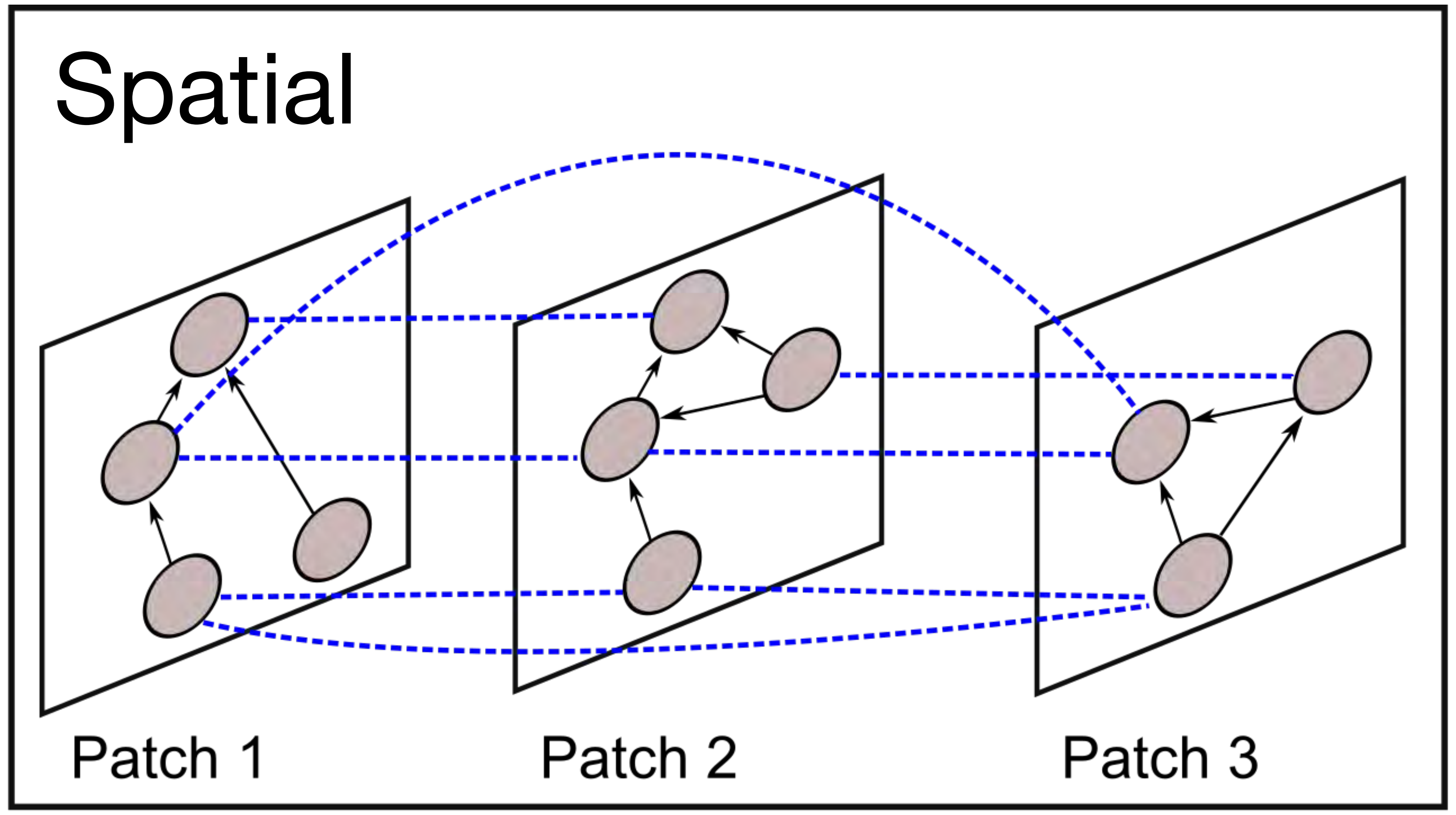
Kéfi et al. 2016

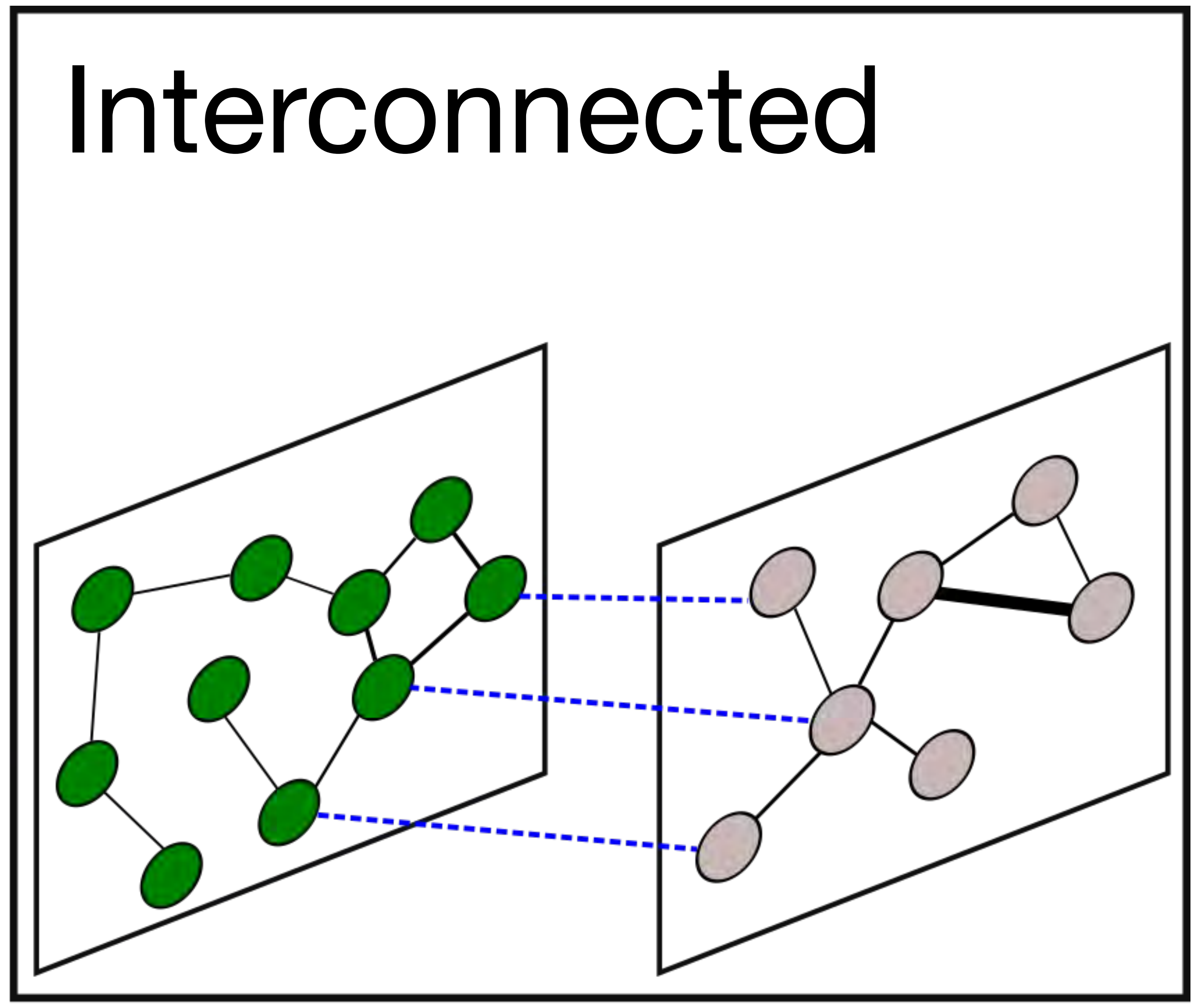


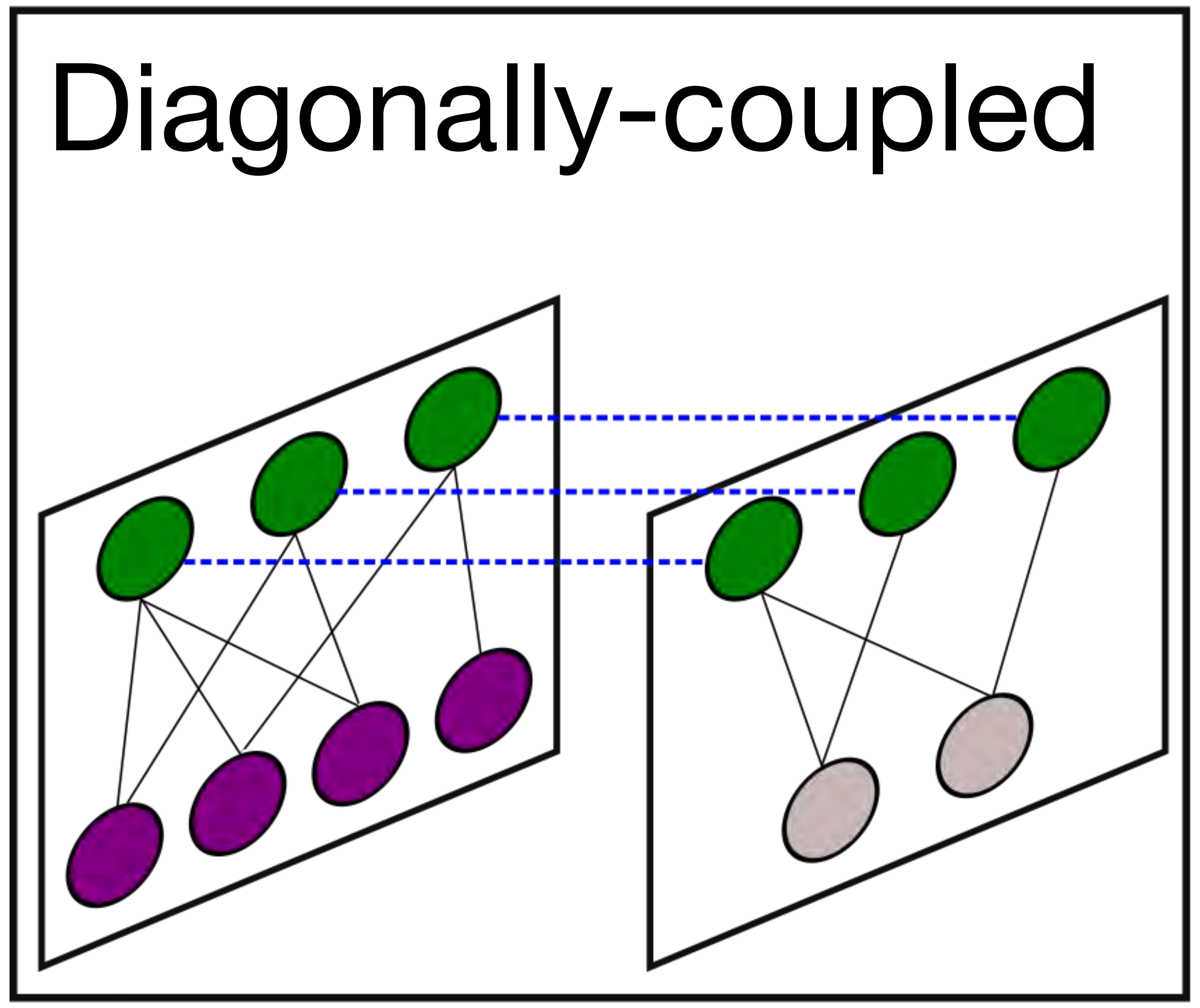
Hagen et al. 2012, Gilarranz et al. 2014

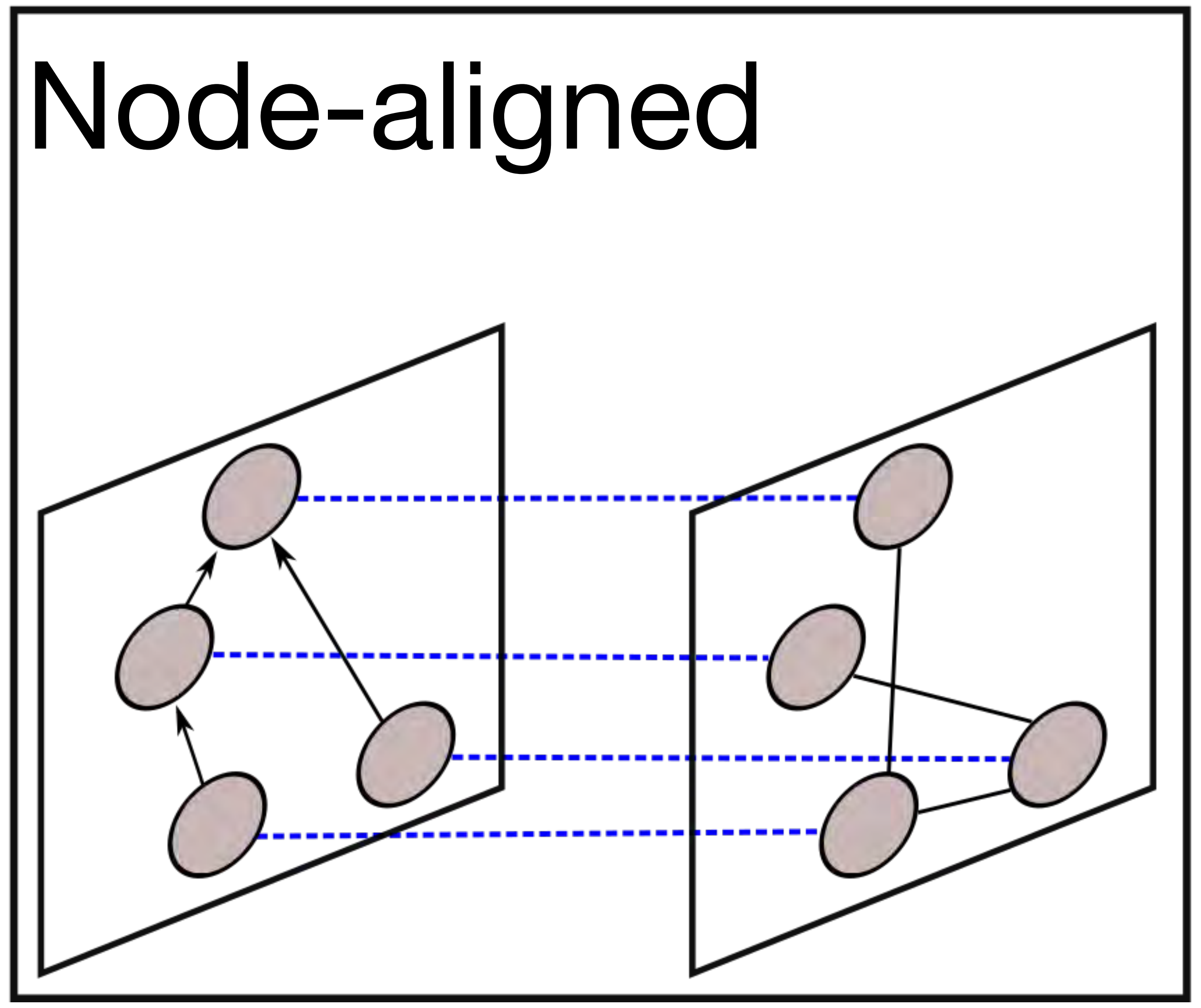
Categories of Ecological Multilayer networks

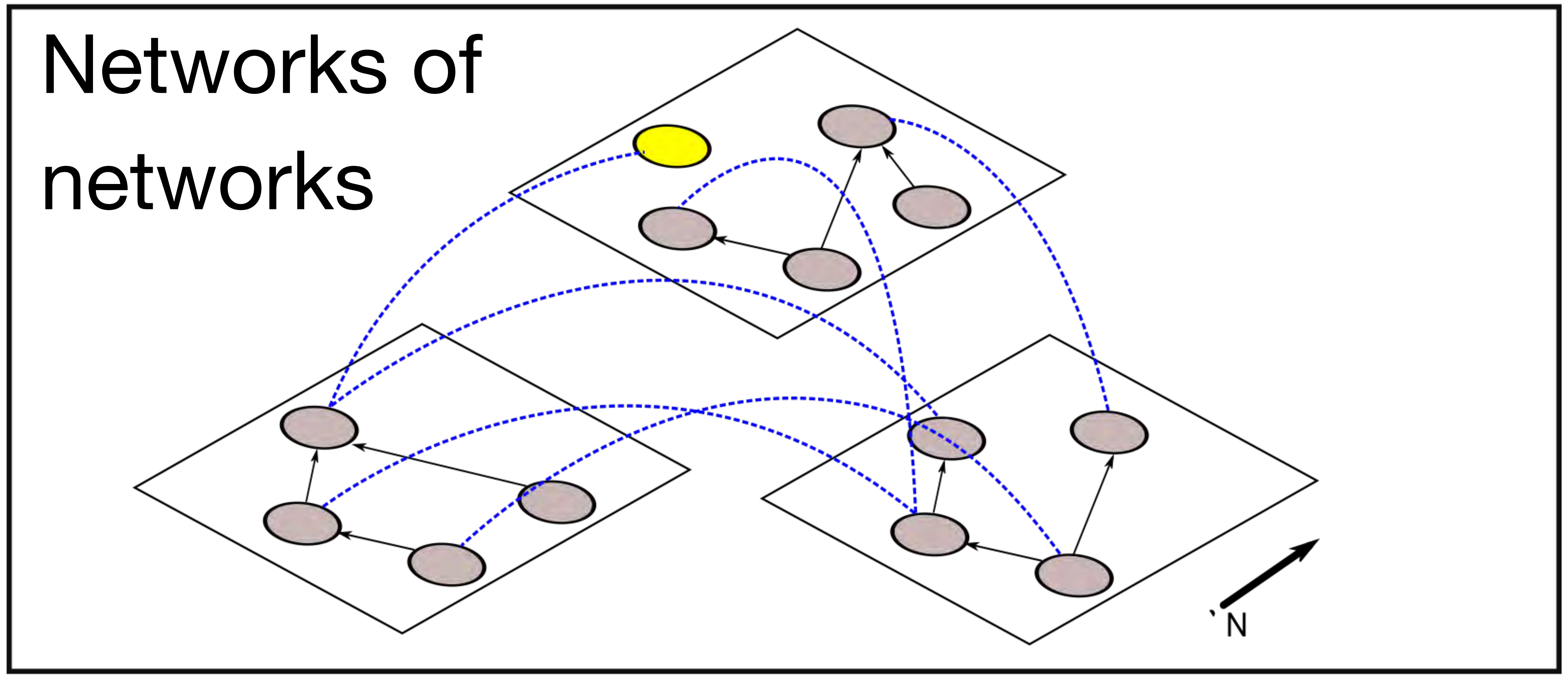






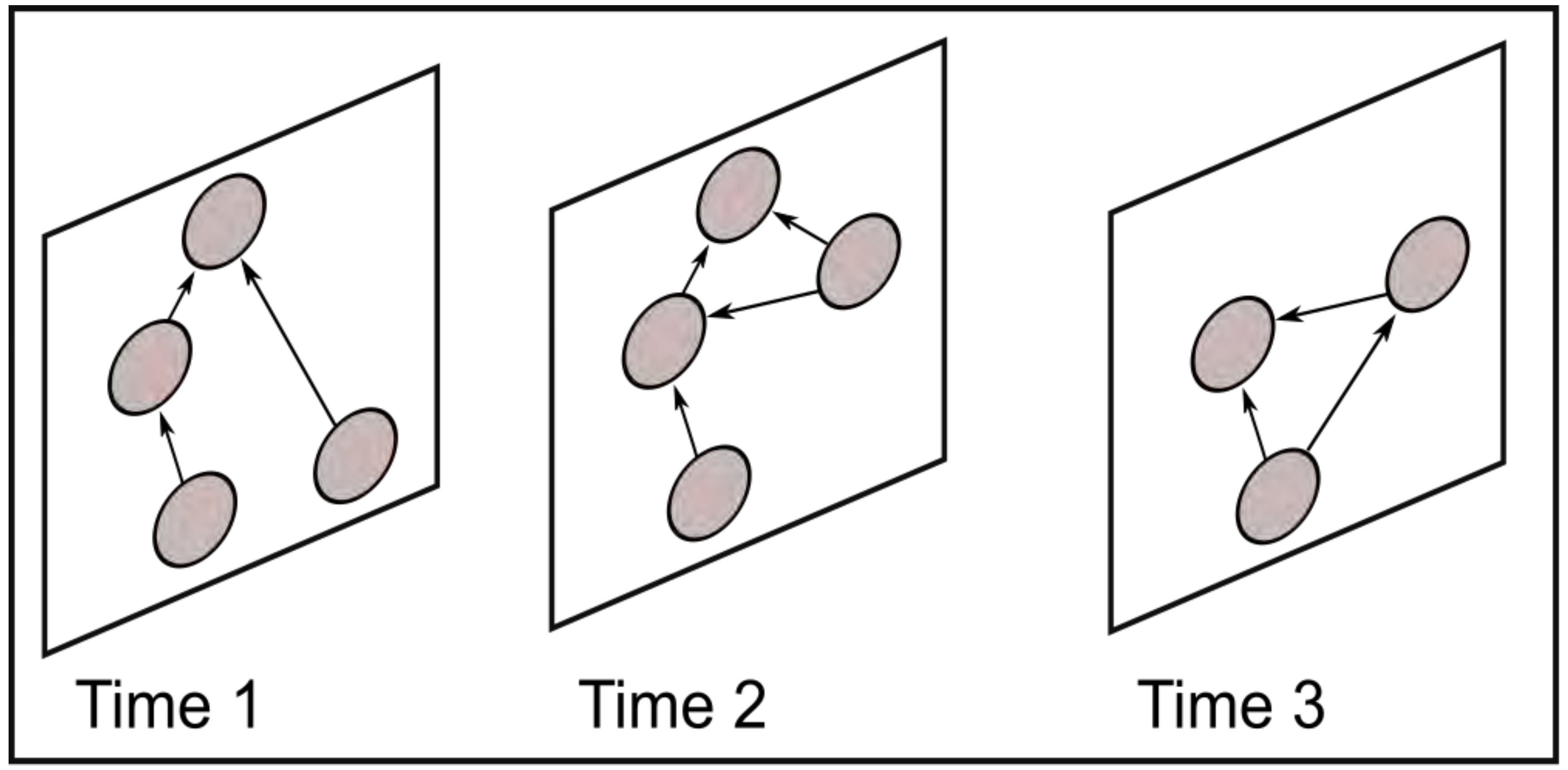




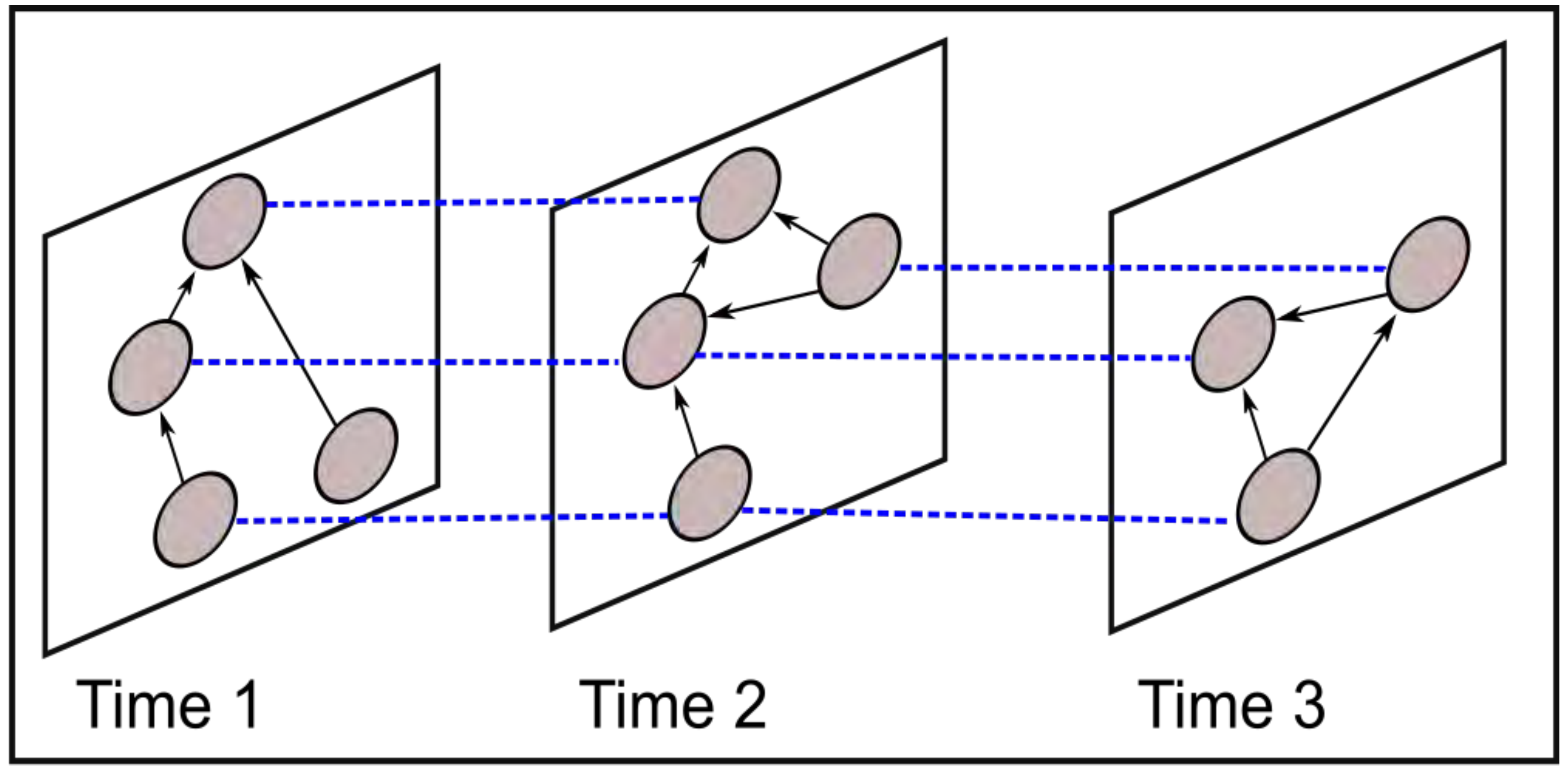


Pilosof et al. 2017, Nature Ecol. Evol.

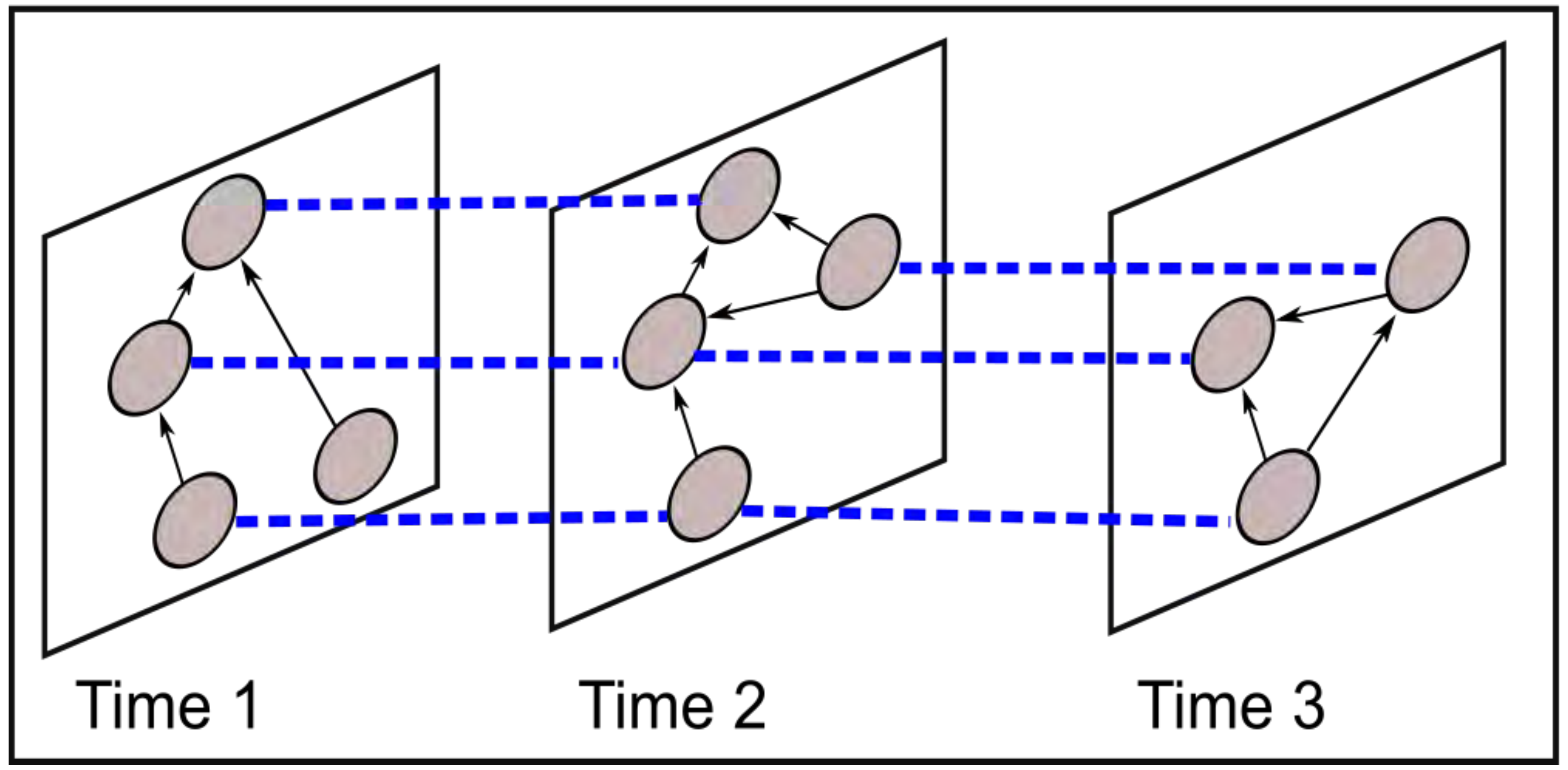
Interlayer edges are the key component



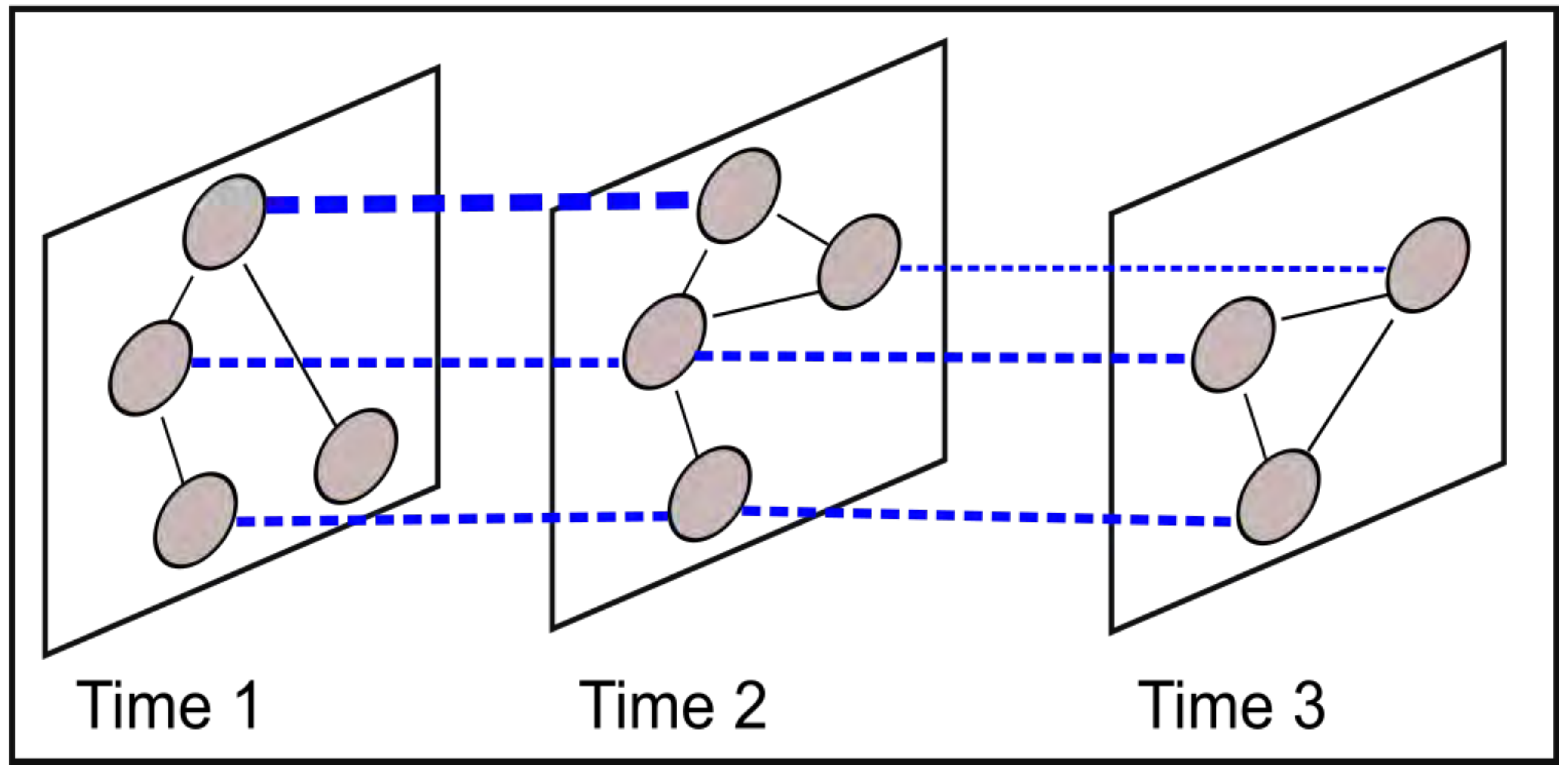
None



Uniform — scale 1

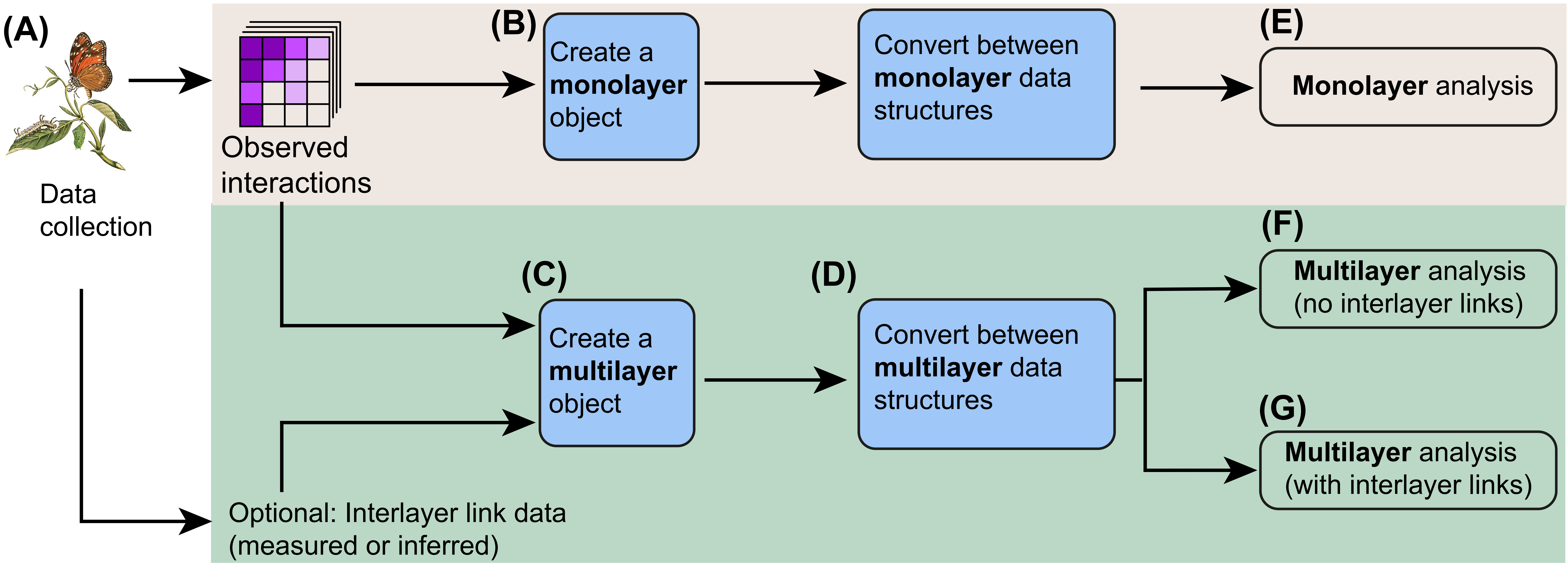


Uniform — scale 2



Non-uniform

Workflow and data organization



Frydman et al 2023, Methods Ecol Evol

APPLICATION

Methods in Ecology and Evolution BRITISH ECOLOGICAL SOCIETY

Identifying flow modules in ecological networks using Infomap

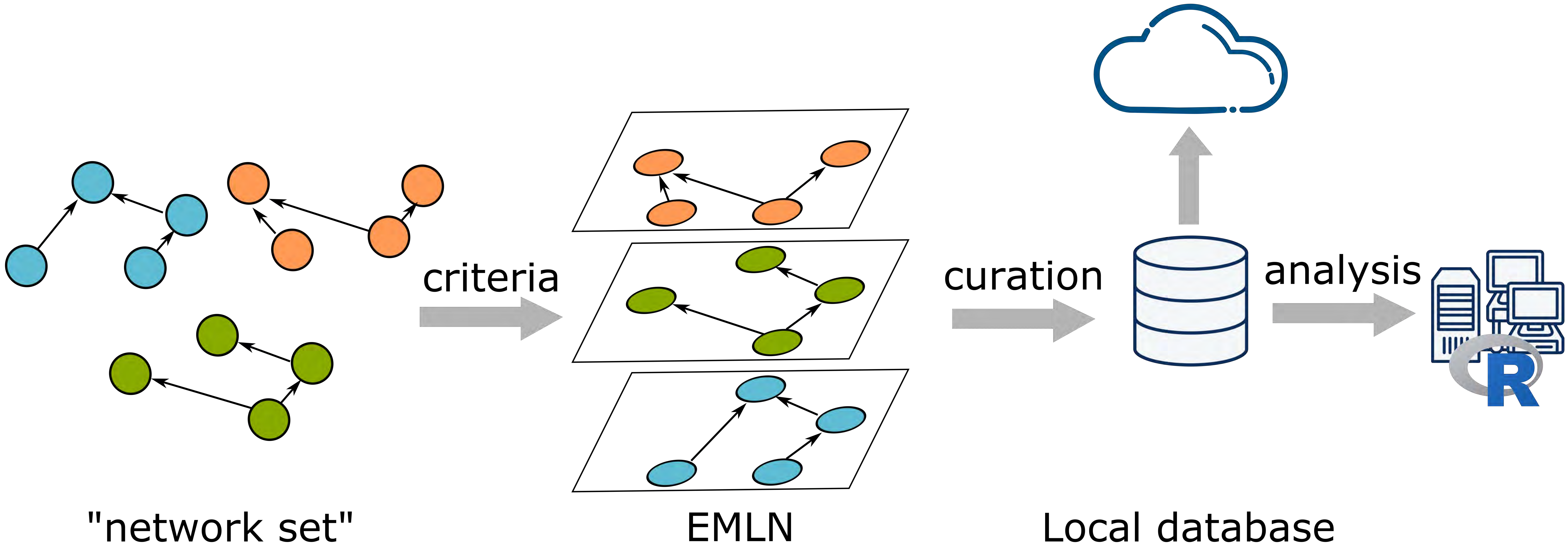
Carmel Farage[1] | Daniel Edler[2,3,4] | Anna Eklöf[5] | Martin Rosvall[2] | Shai Pilosof[1]

- Community detection
- Organize and analyze EMLNs

Martin Rosvall

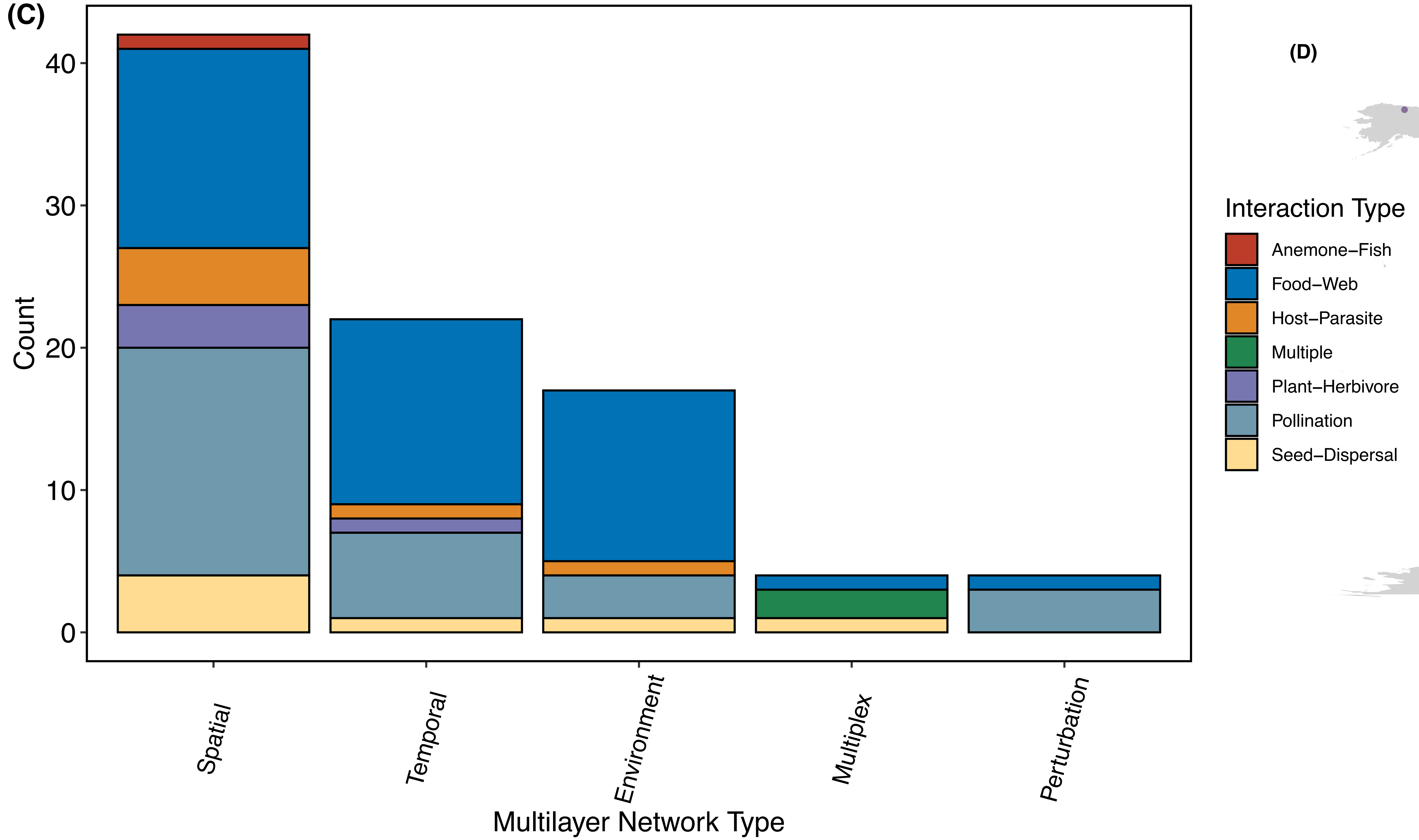
Anna Eklöf

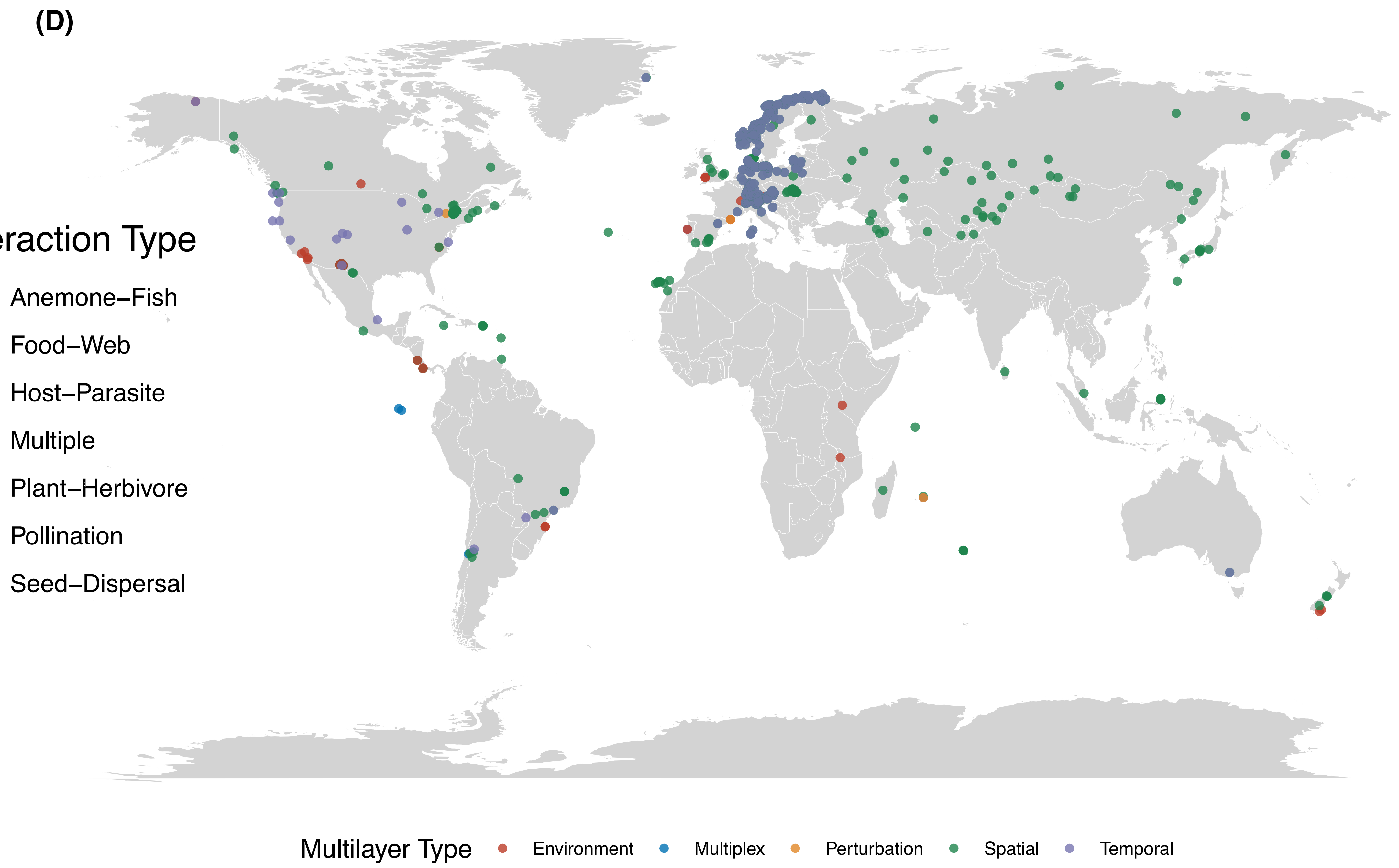
EMLN database



Frydman et al 2023, *Methods Ecol Evol*

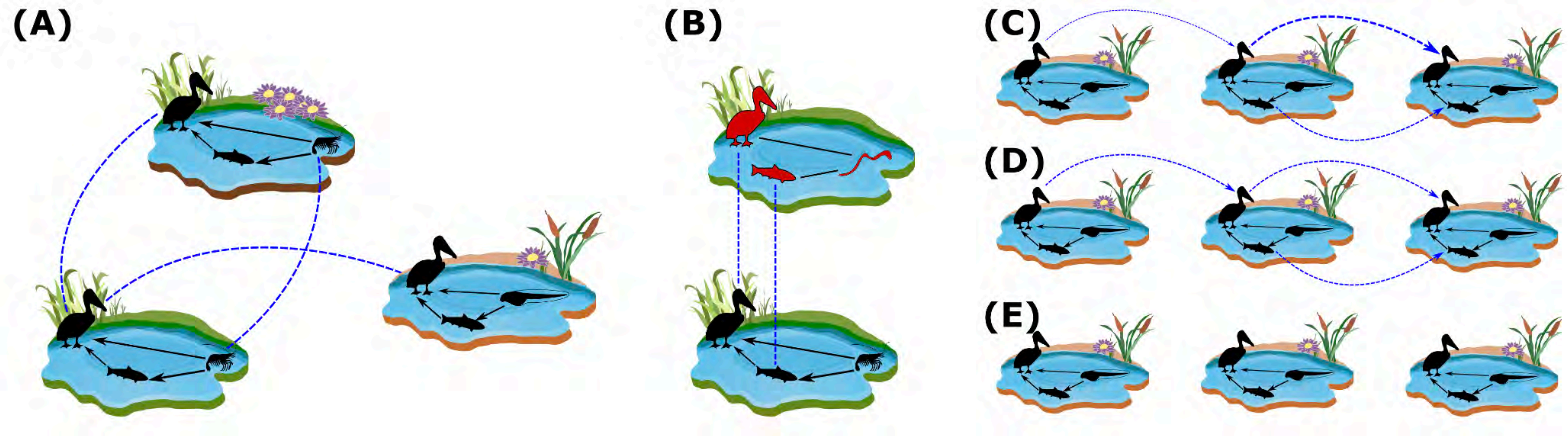
EMLN database





Frydman et al 2023, Methods Ecol Evol

1. None.
2. Inferred.
3. Measured.

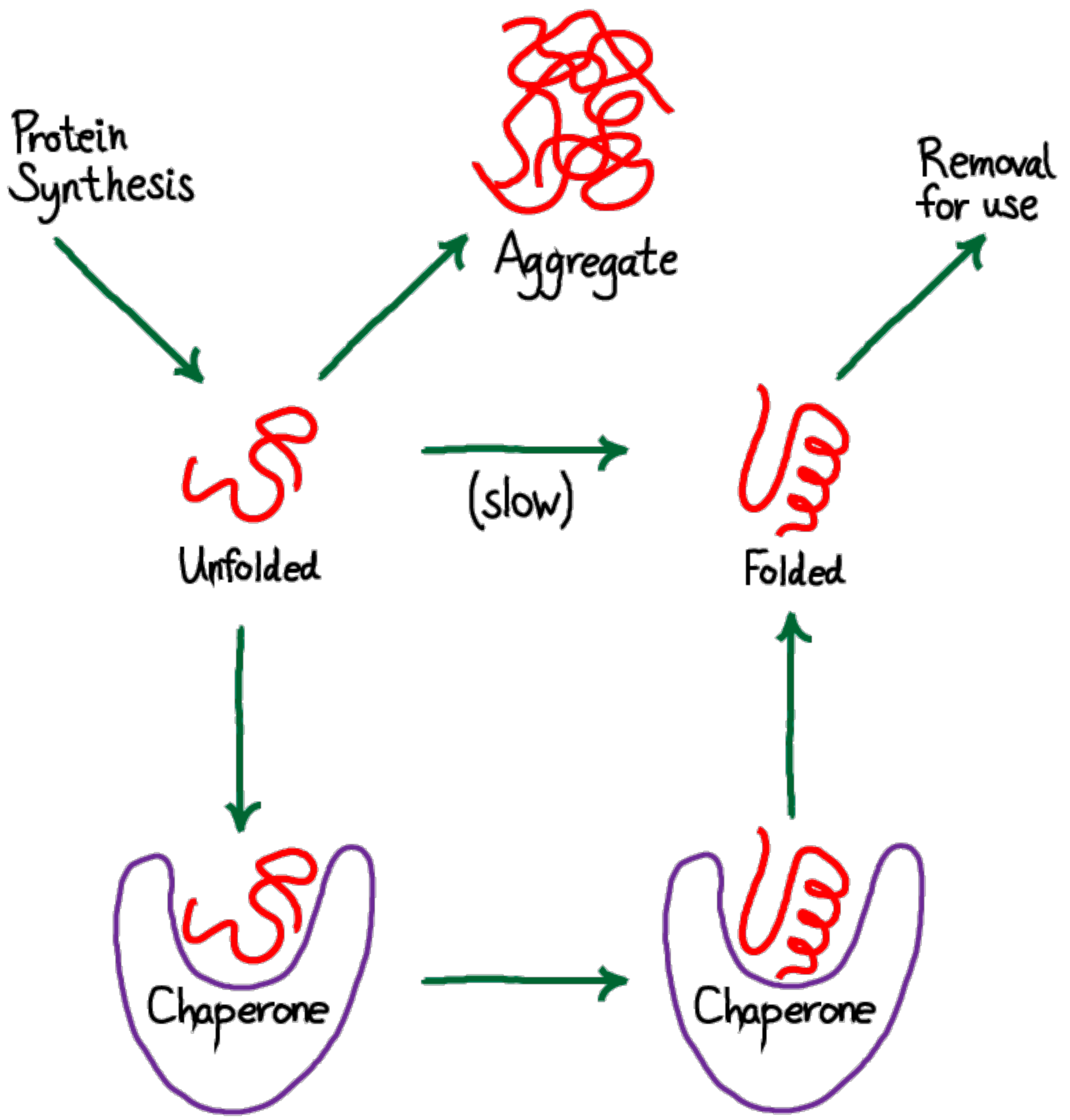


Constructing interlayer links

1. Are they needed?
2. What is the hypothesis encoded?
3. Intralayer vs interlayer number and weight distribution.
4. Algorithms and tools applied.

Protein interactions in changing cancer environments

- Proteins are folded by chaperones.
- Can we predict **chaperone-client interactions** across cancer environments?
- What are the network-wide consequences of targeting proteins?




Geut Galai


Prof. Barak Rotblat

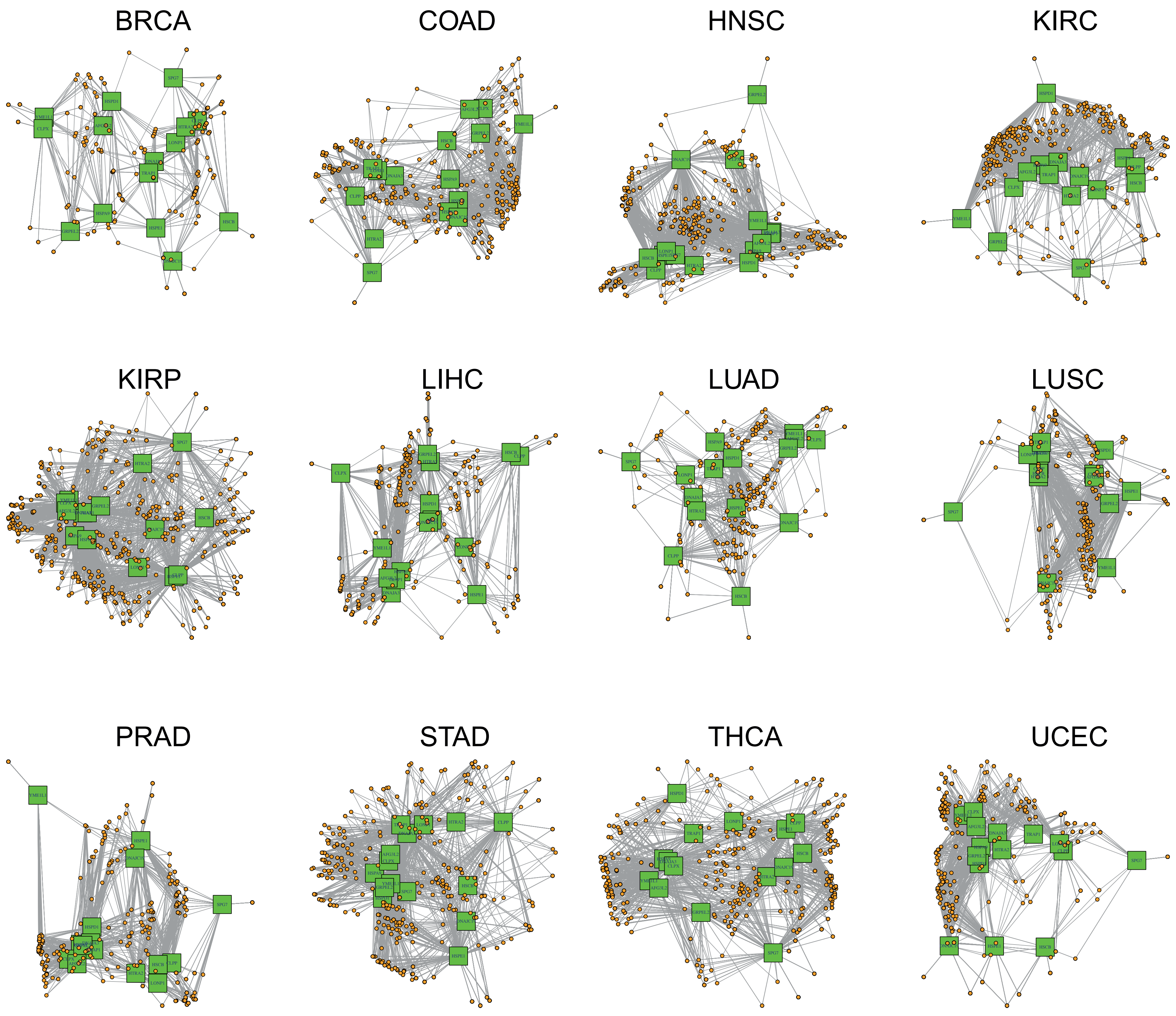

Xie He

Galai, He, Rotblat, Pilosof, Nature Communications 2023

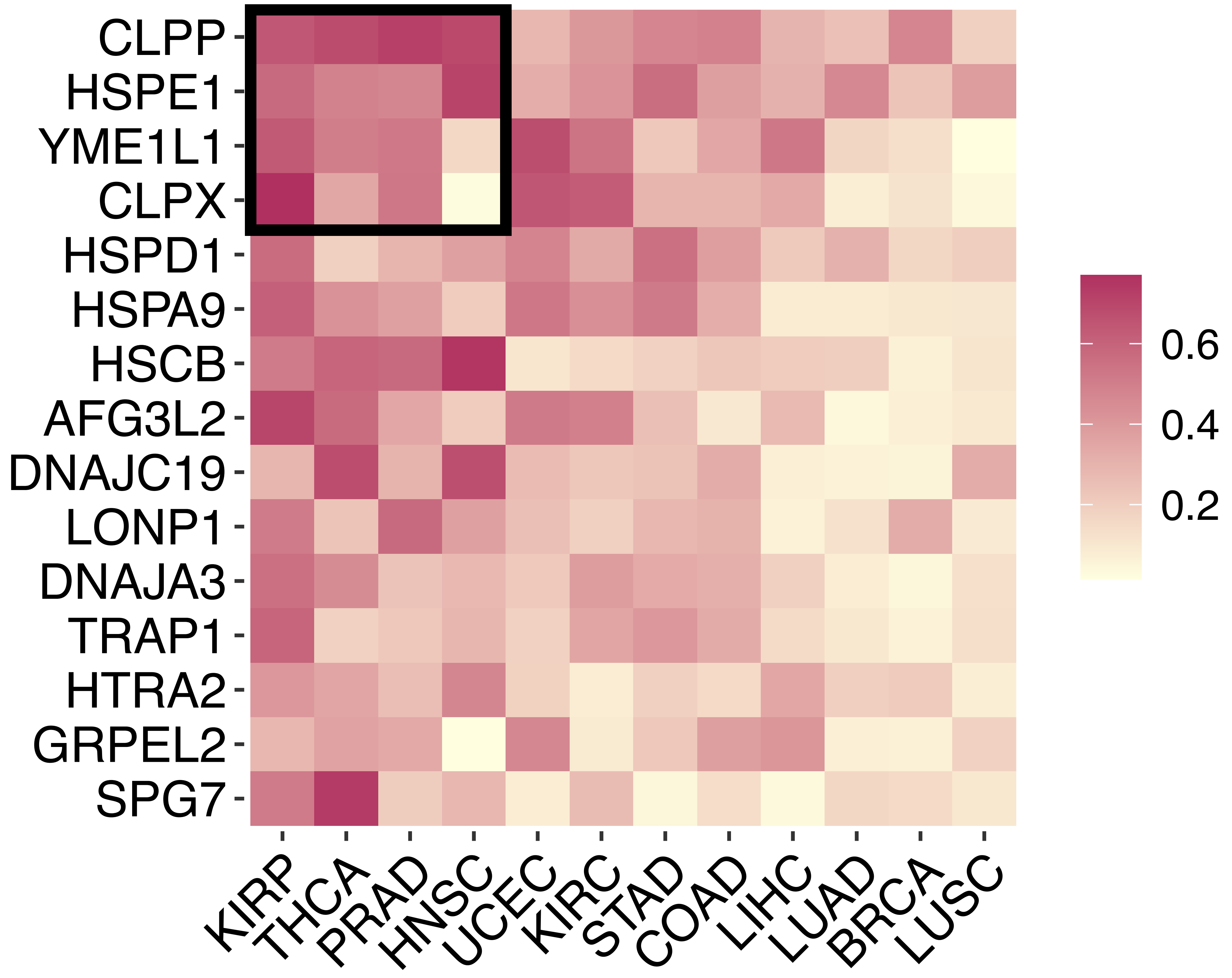
Protein interactions in changing cancer environments

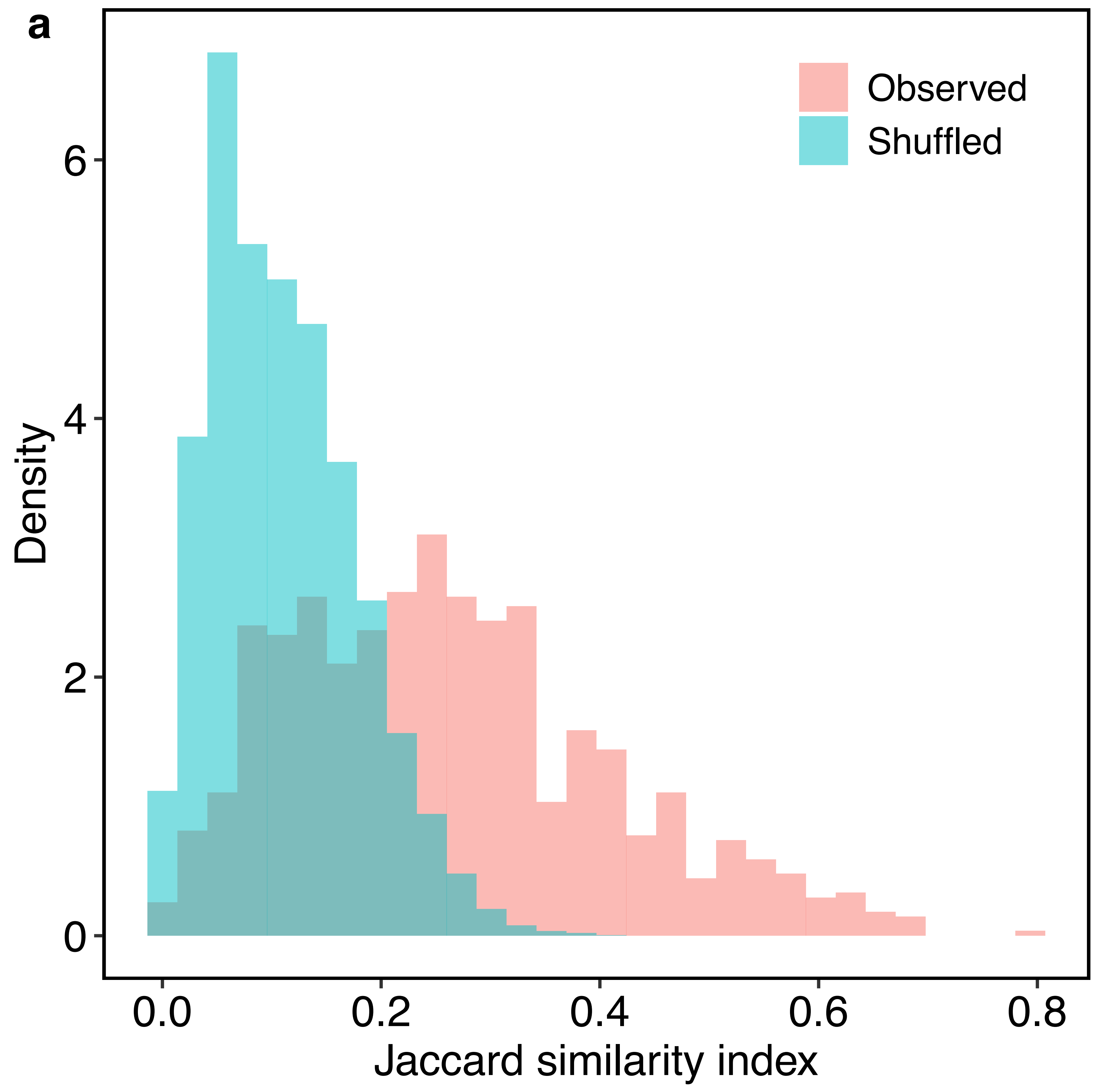
How do chaperone-client interactions vary across cancer environments?

What are the network-wide consequences of targeting proteins?



Galai, He, Rotblat, Pilosof, Nature Communications 2023





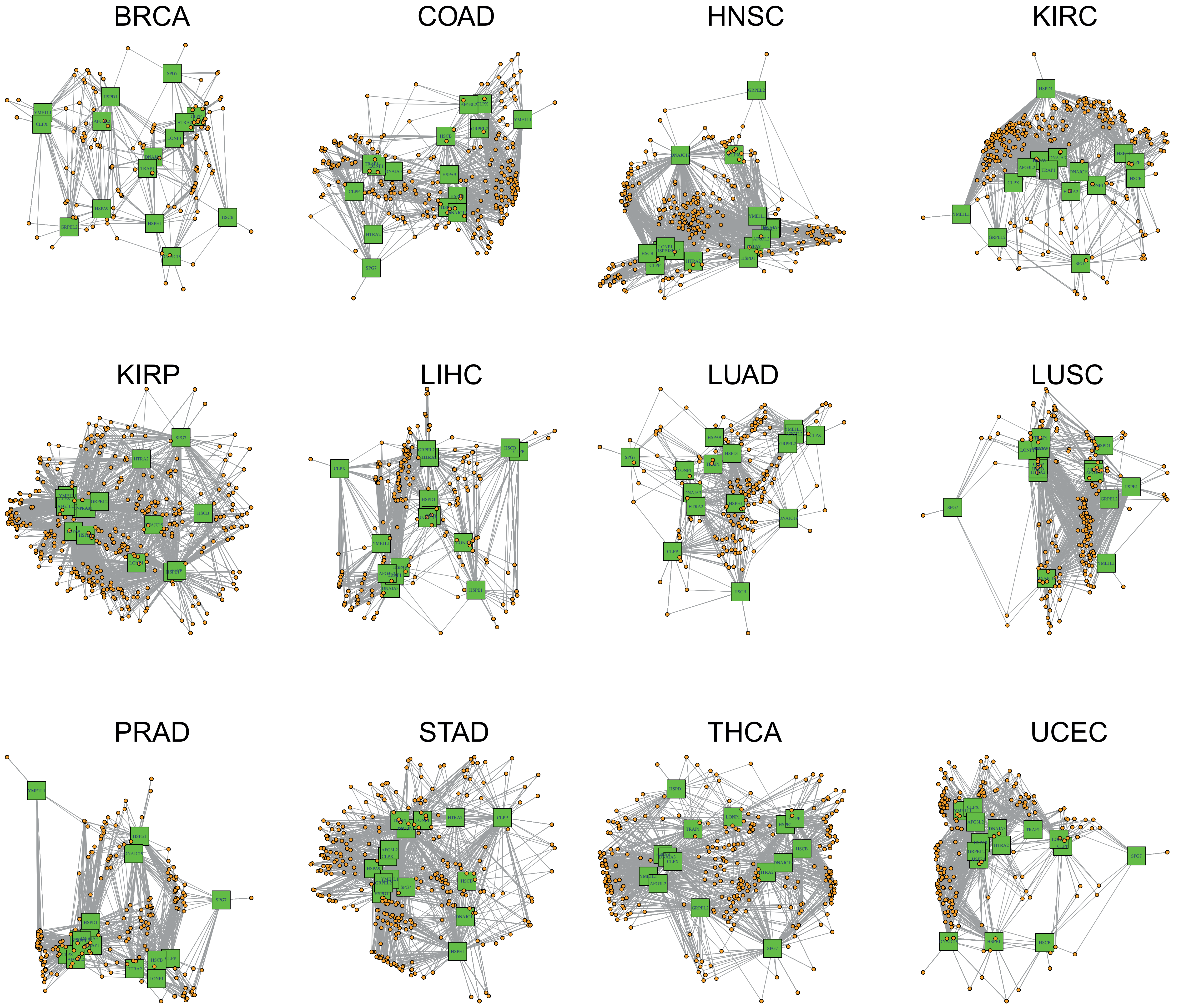
Non-random patterns in chaperone realized niche across cancer environments.

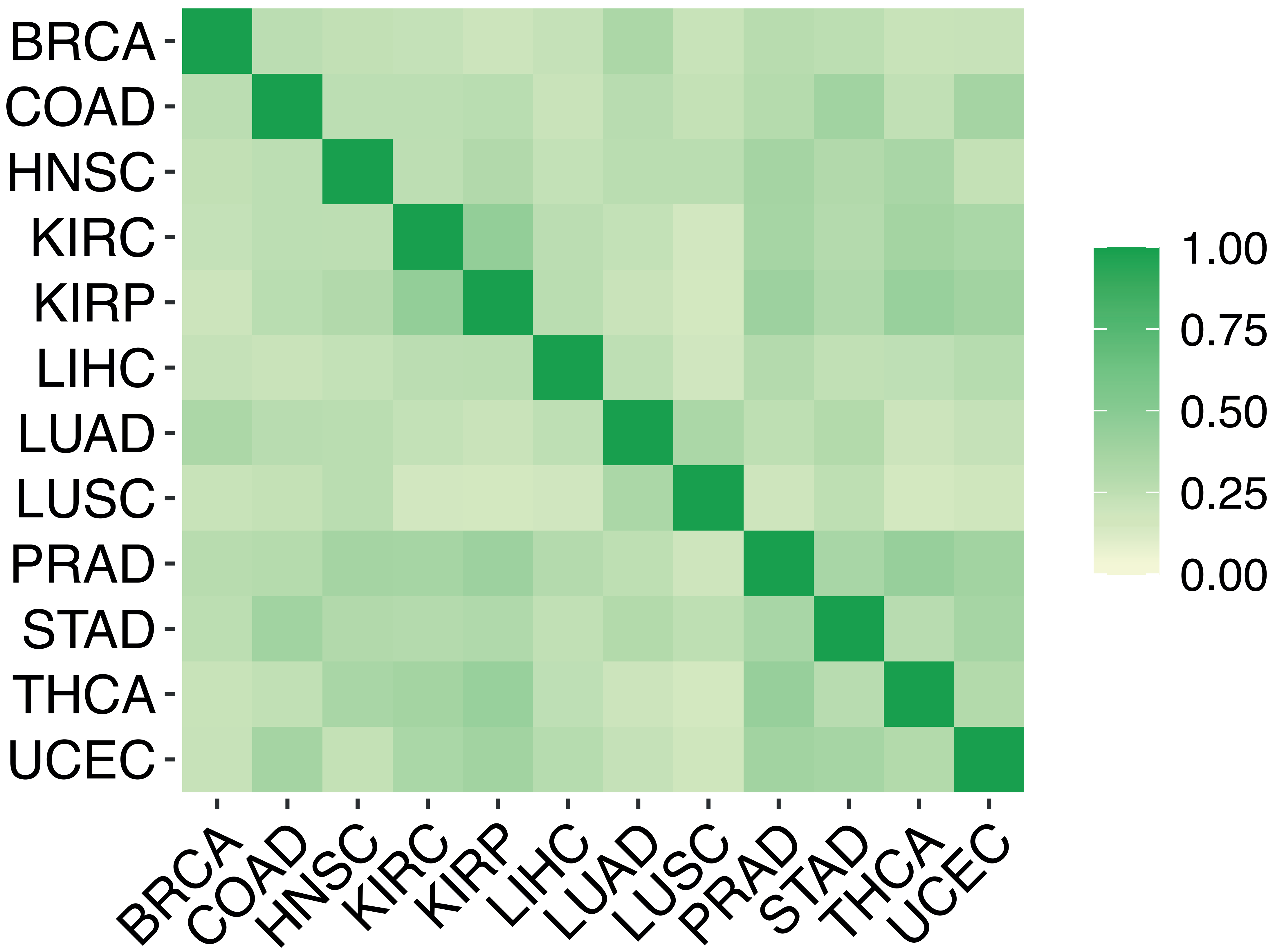
$$R_c^\alpha = L_c^\alpha / P_c$$

Chaperones share clients across cancers more than expected at random

Galai, He, Rotblat, Pilosof, Nature Communications 2023

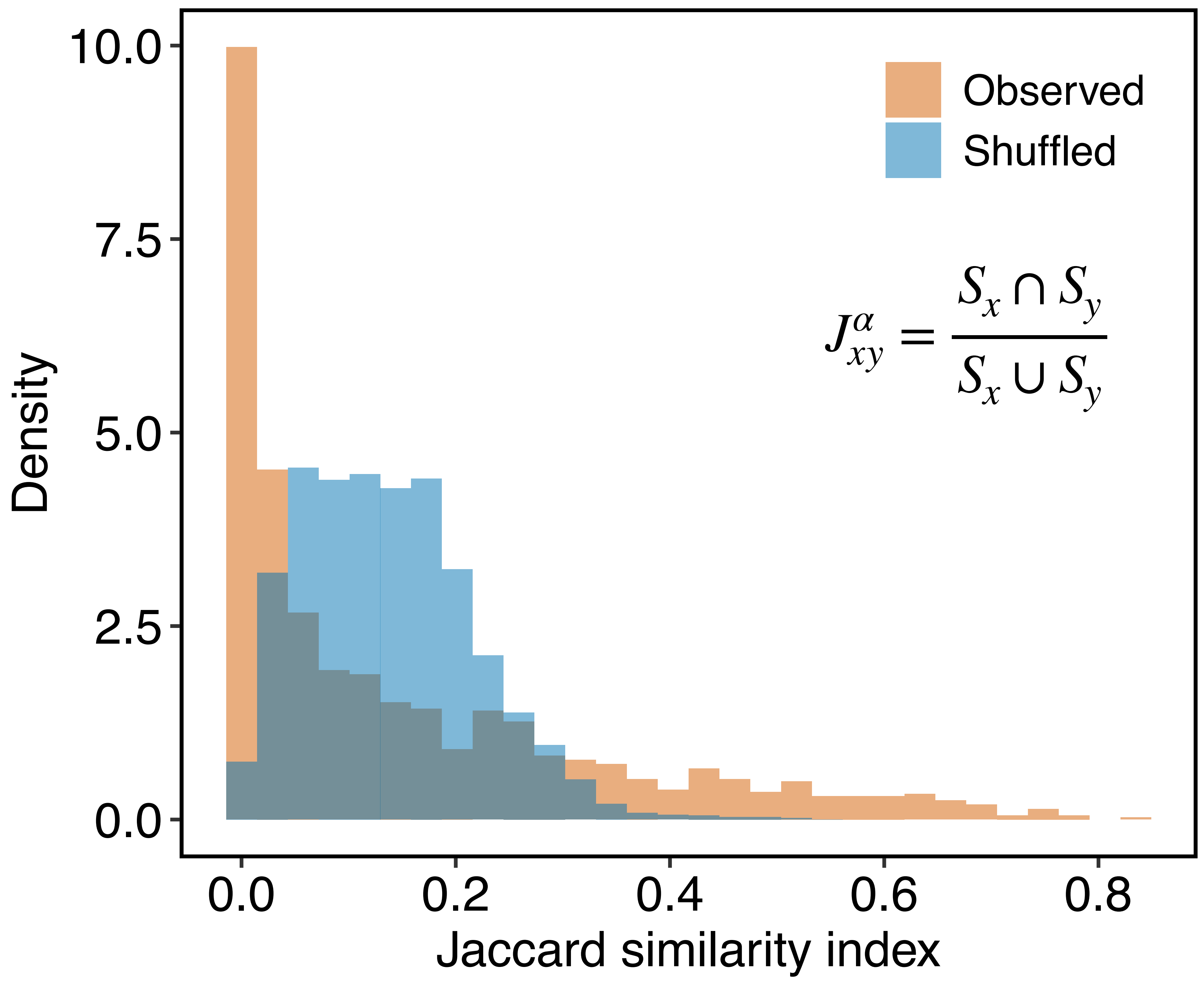
Low similarity in interactions across cancers

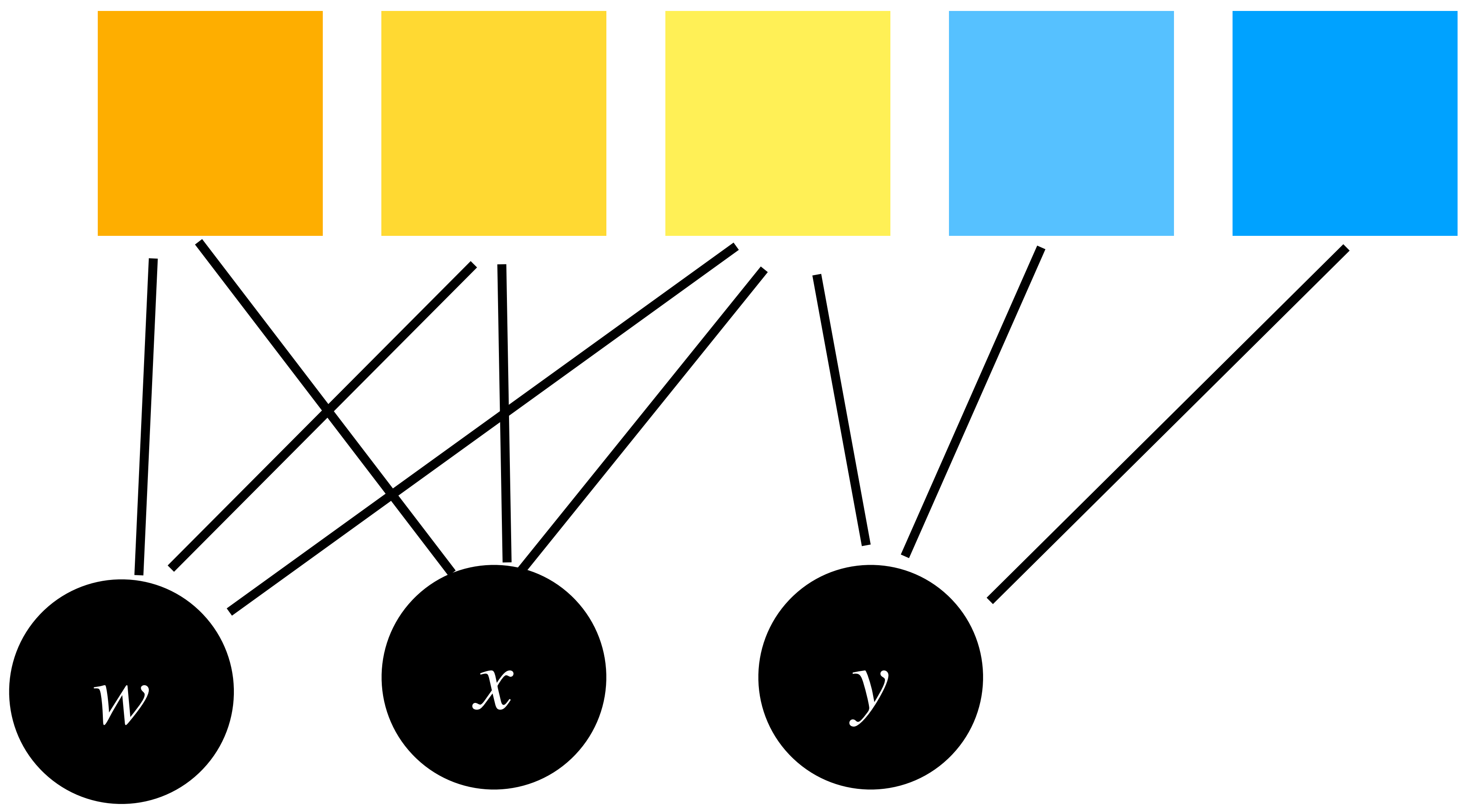


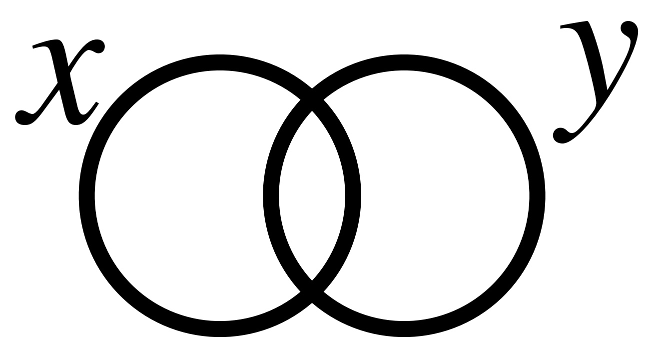


SBM model from: De Bacco et al 2017, PRE

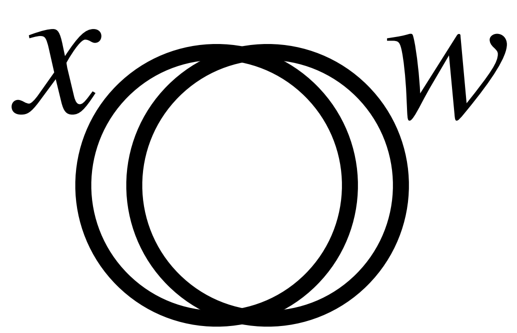
Chaperones have strong niche differentiation but some redundancy





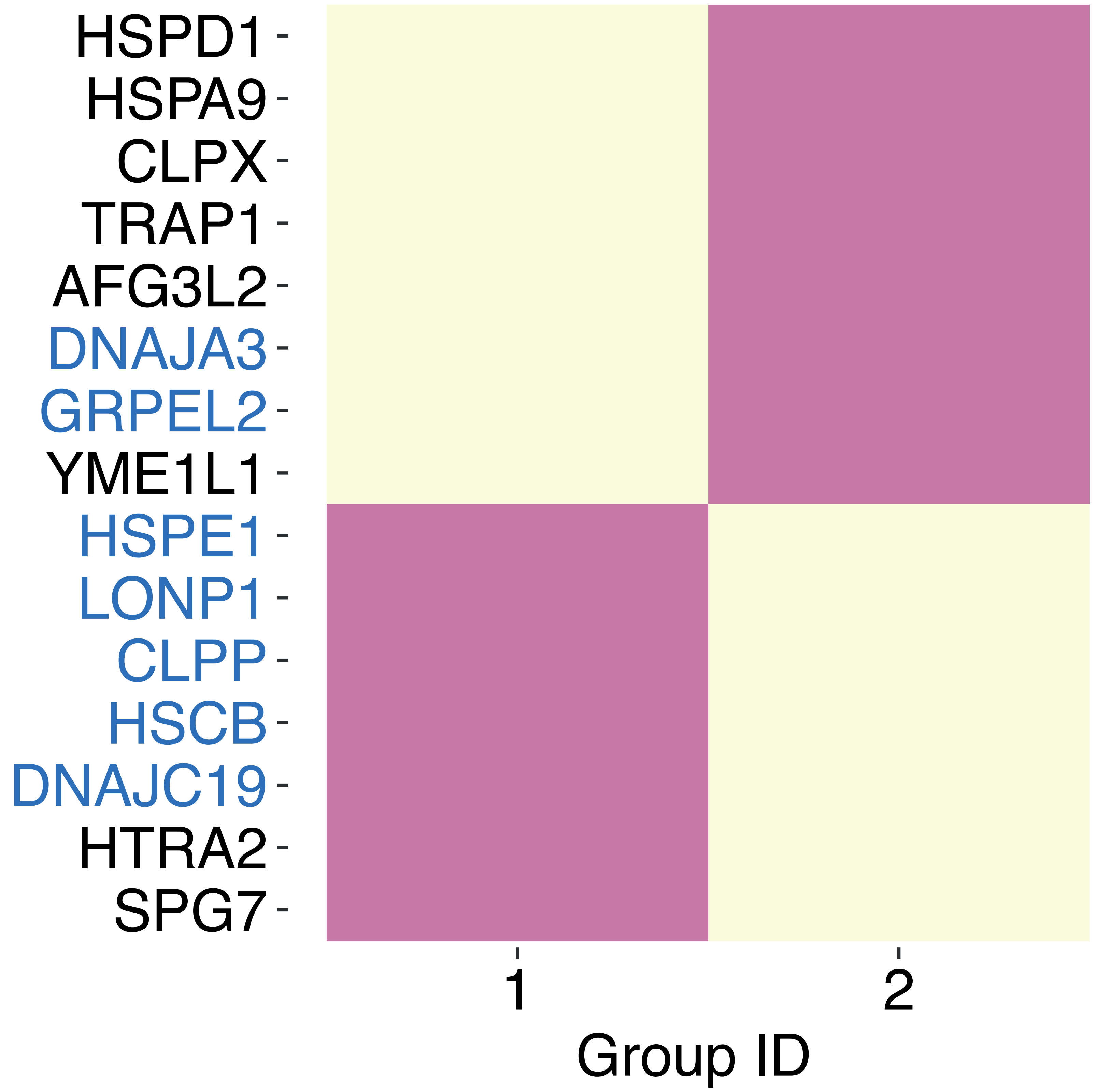


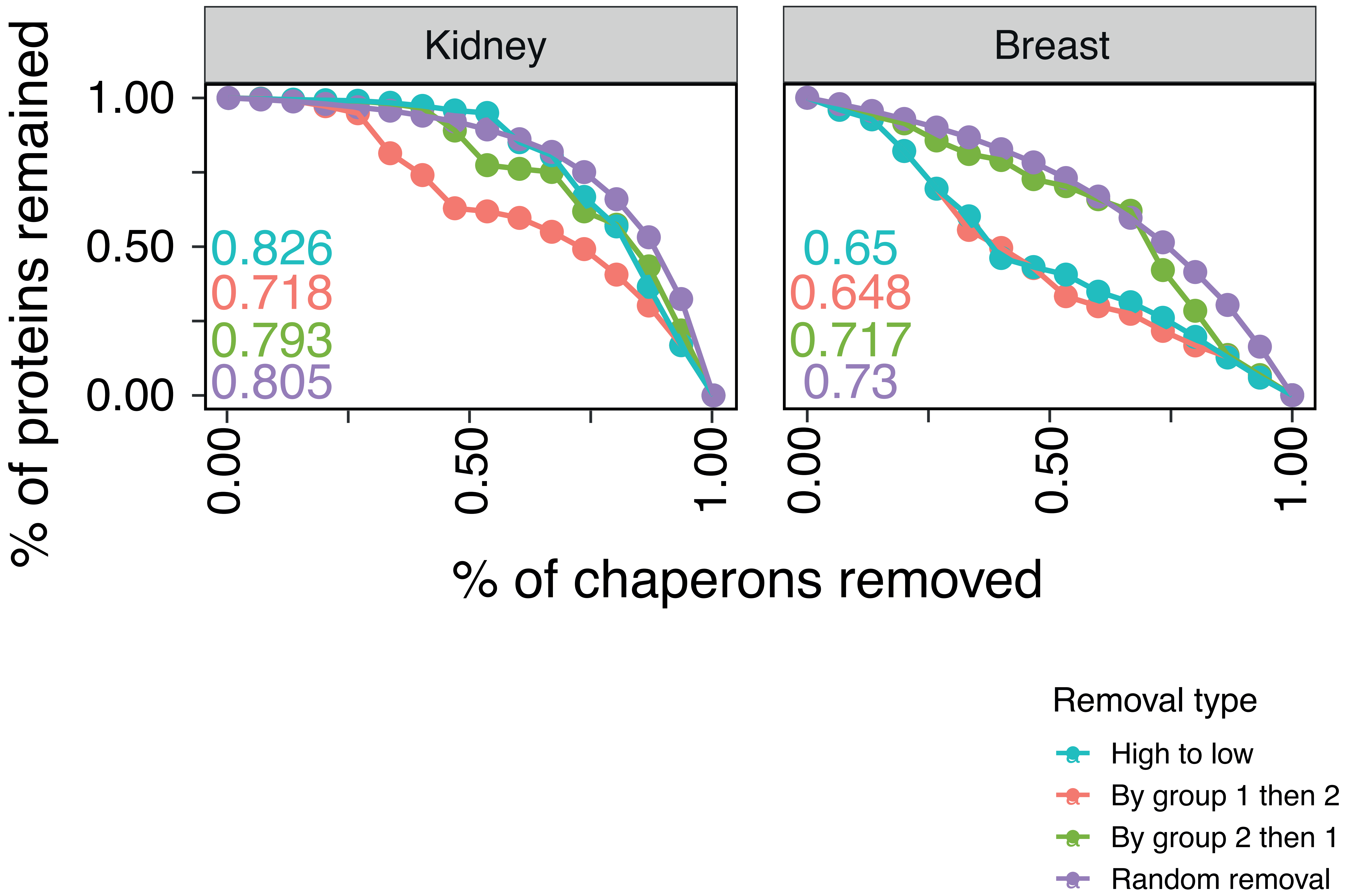
Niche separation



Redundancy

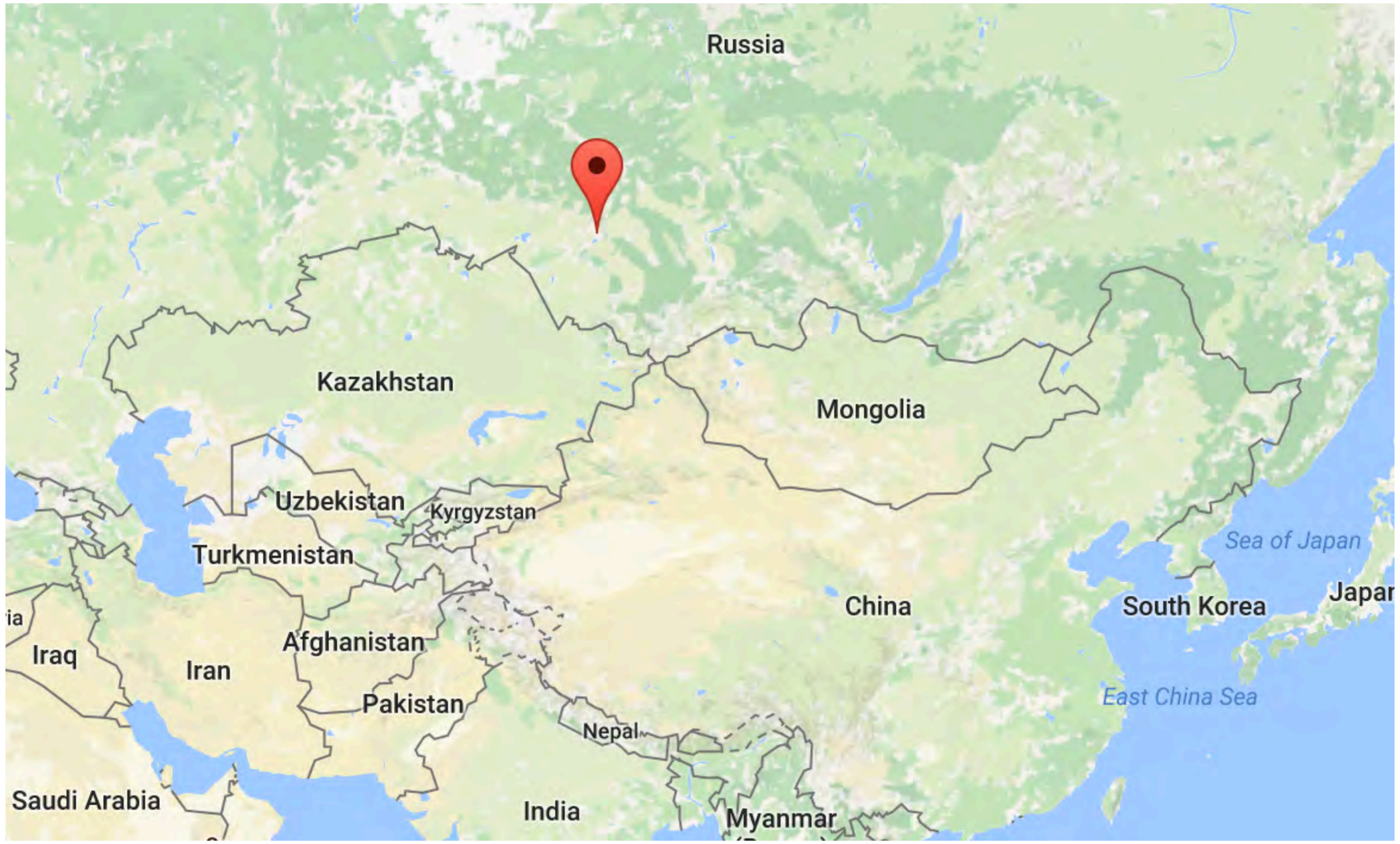
Chaperones are organized in two groups, affecting robustness

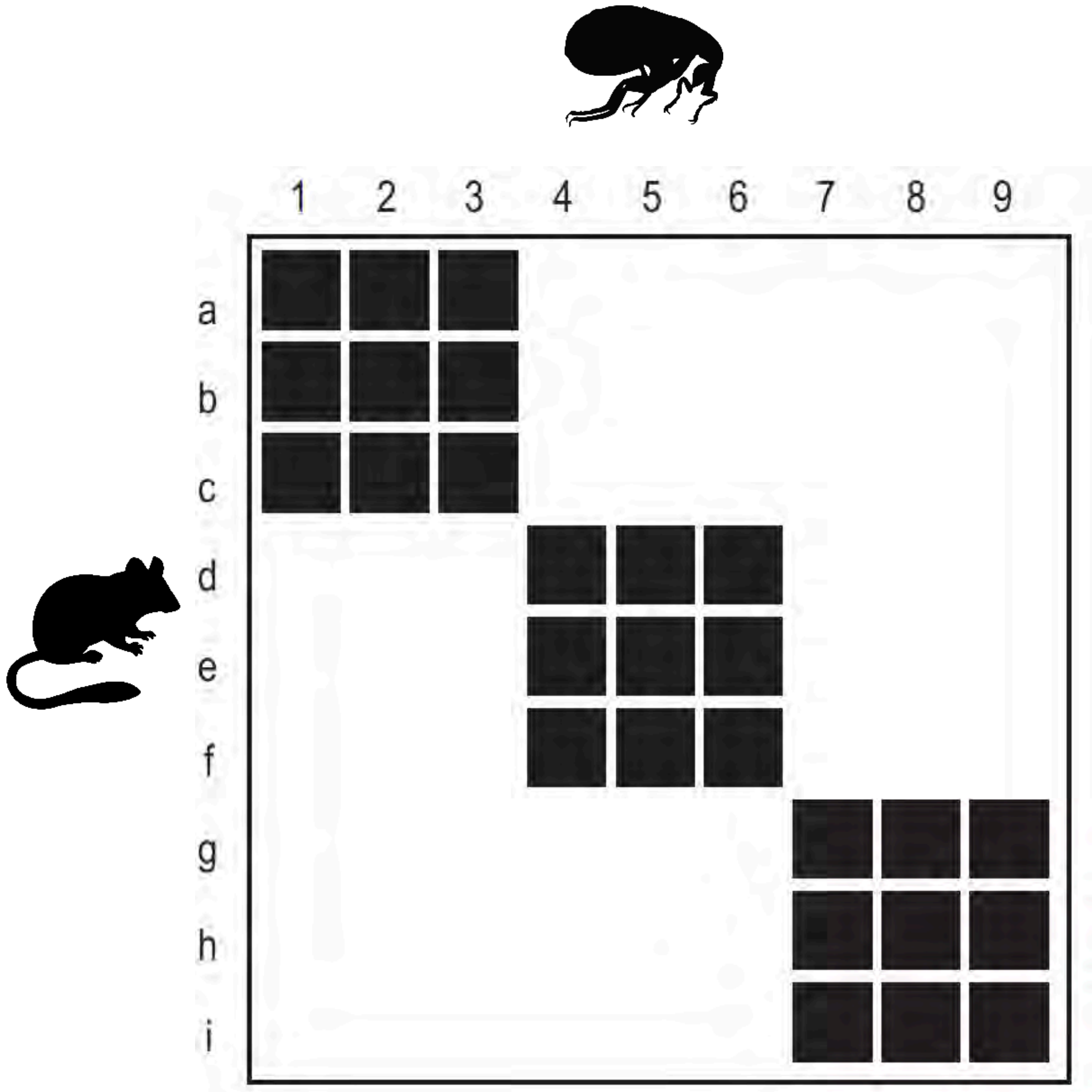


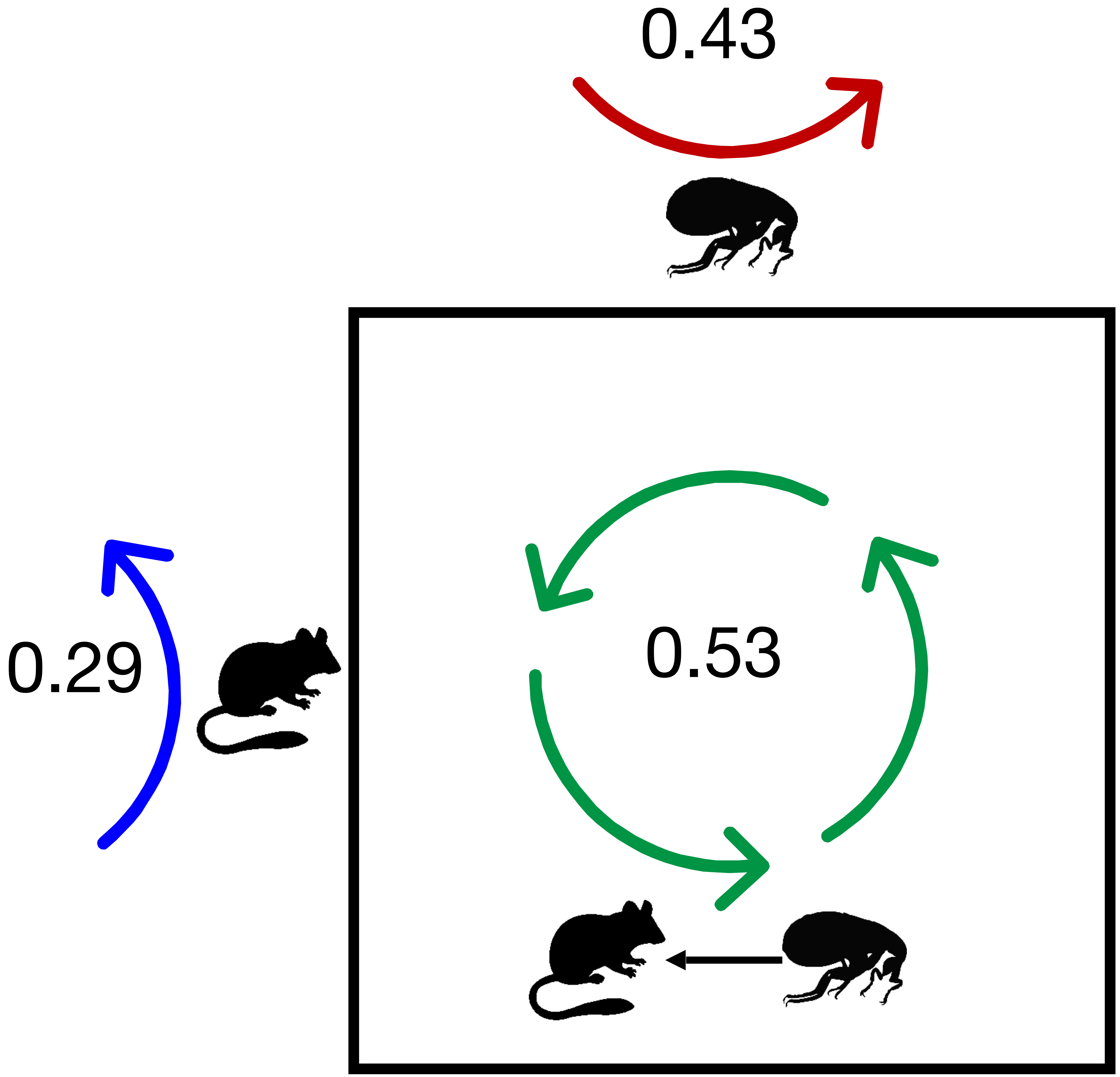


SBM model from: De Bacco et al 2017, PRE

- 6 consecutive summers (1982-1987).
- 22 mammalian hosts.
- 56 ectoparasites (fleas and mites).

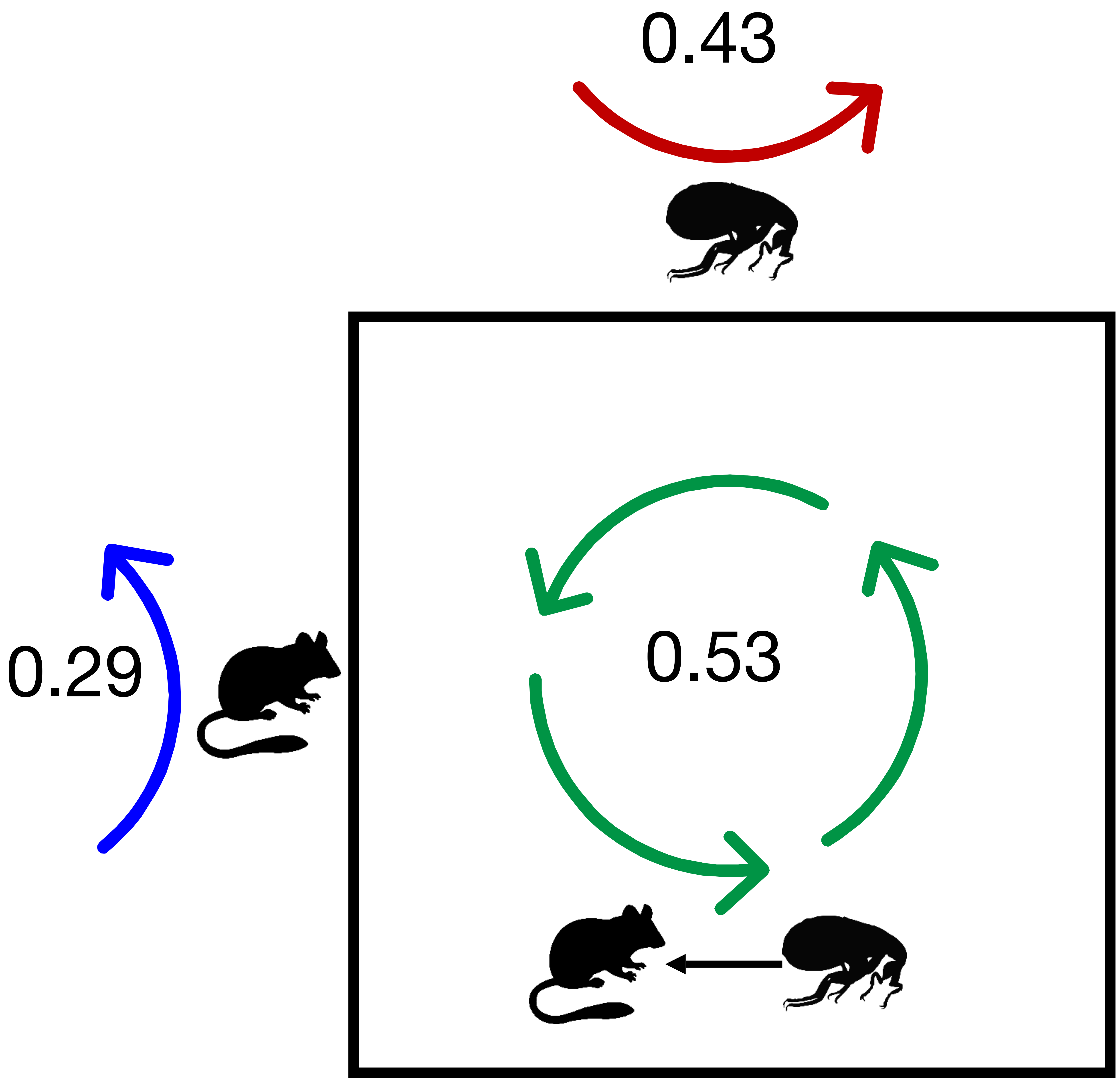




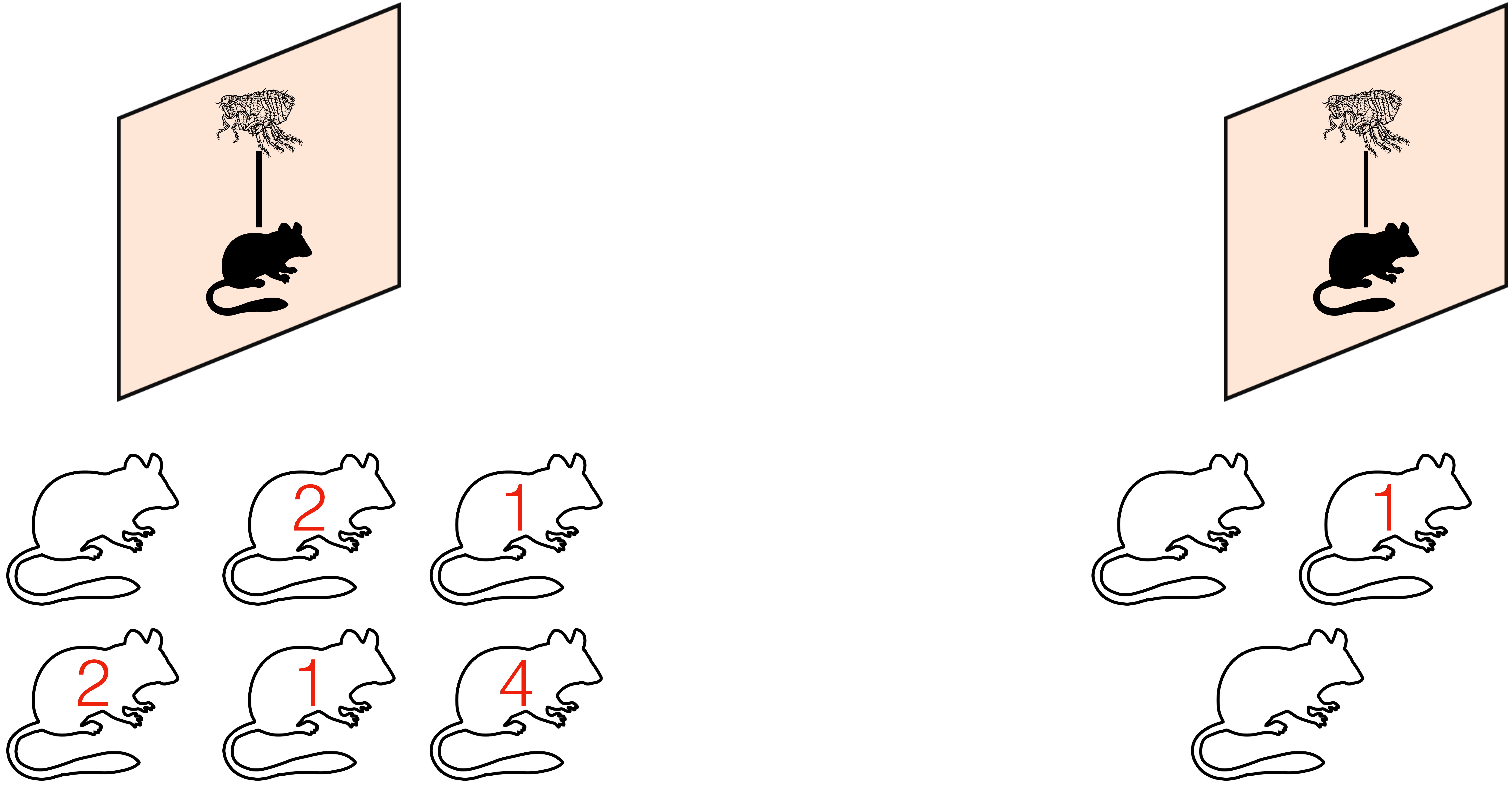


Pilosof et al. 2013, J. Anim Ecol.

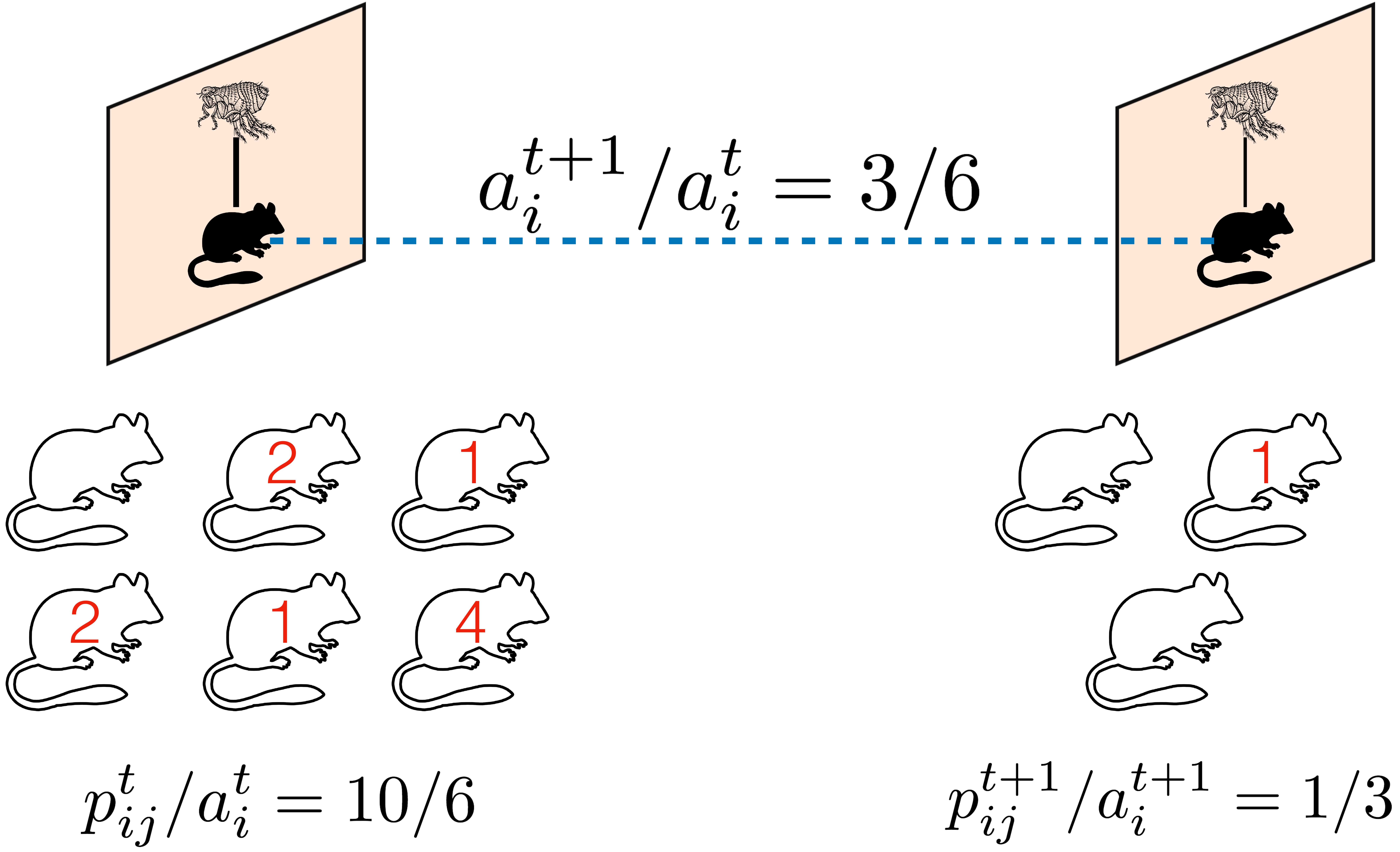
Does the structure of the host-parasite community evolve in time?



Pilosof et al. 2013, J. Anim Ecol.

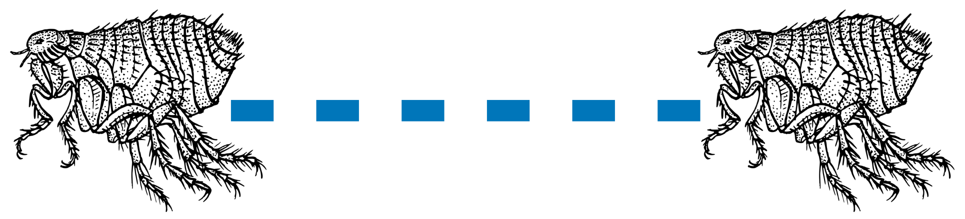


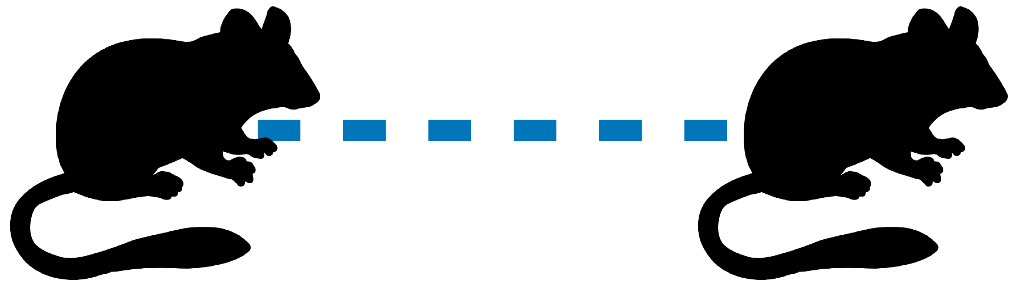
$$p_{ij}^{t}/a_{i}^{t} = 10/6 \qquad p_{ij}^{t+1}/a_{i}^{t+1} = 1/3$$


$a_i^{t+1}/a_i^t = 3/6$
2
1
2
1
4
1
$p_{ij}^t/a_i^t = 10/6$
$p_{ij}^{t+1}/a_i^{t+1} = 1/3$

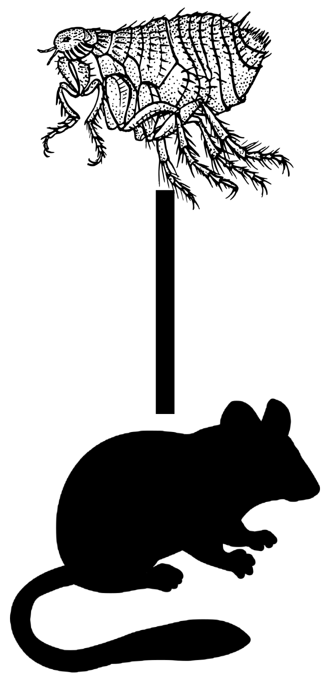
A future <u>intra-</u>layer interaction depends on past <u>inter-</u>layer interactions

$$\begin{cases} \boxed{n_i^{t\to t+1}} = \dfrac{p_i^{t+1}}{p_i^t} \Rightarrow p_i^{t+1} = p_i^t \cdot n_i^{t\to t+1} \\ \\ \boxed{n_j^{t\to t+1}} = \dfrac{a_j^{t+1}}{a_j^t} \Rightarrow a_j^{t+1} = a_j^t \cdot n_j^{t\to t+1} \end{cases}$$

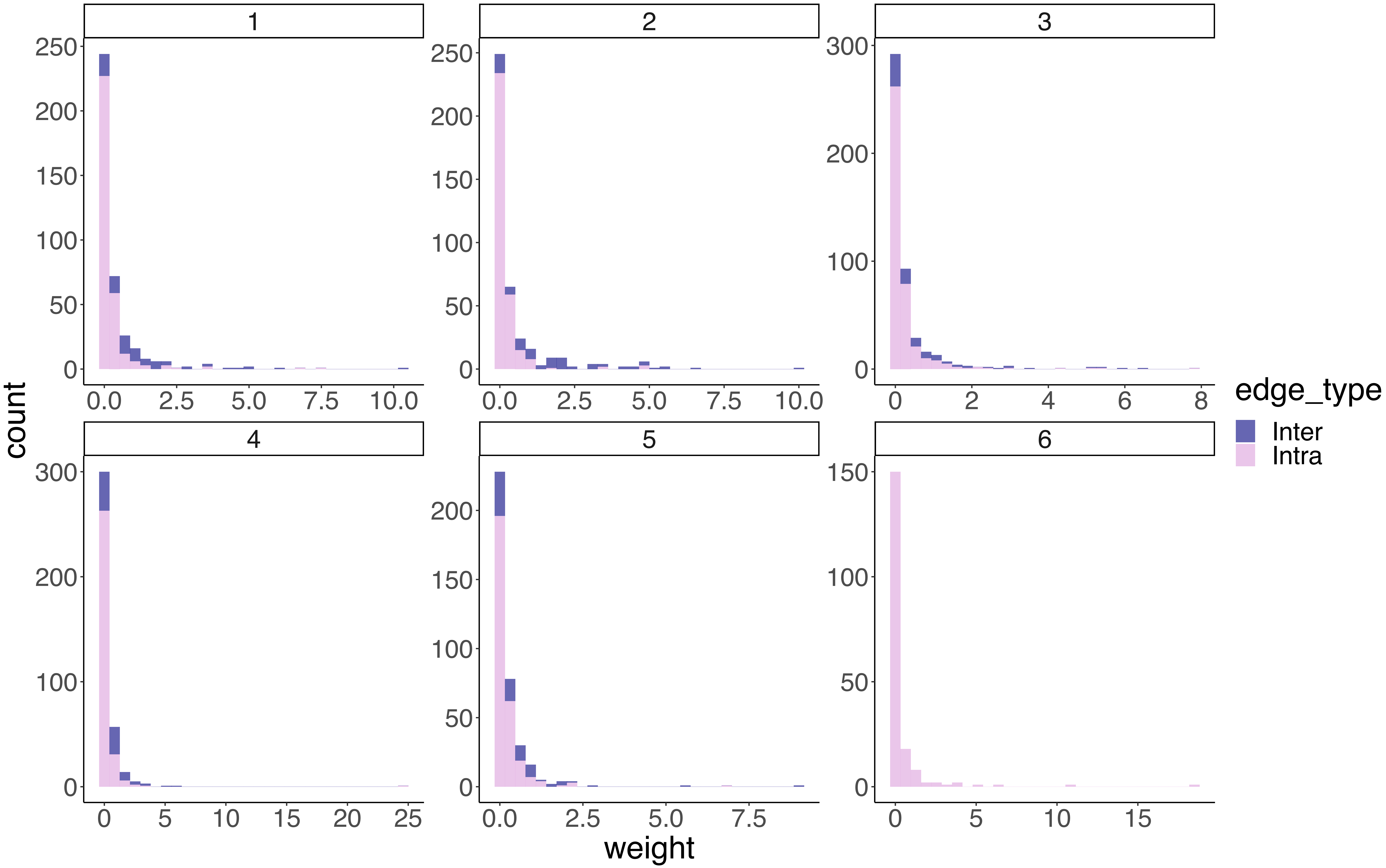


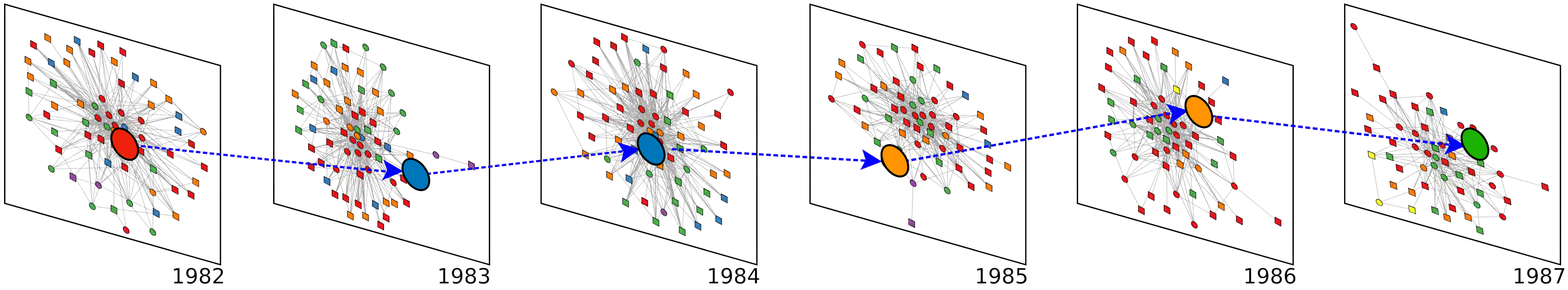


$$m_{ij}^{t+1} = \frac{p_i^{t+1}}{a_j^{t+1}} = \frac{p_i^t \cdot \boxed{n_i^{t\to t+1}}}{a_j^t \cdot \boxed{n_j^{t\to t+1}}}$$



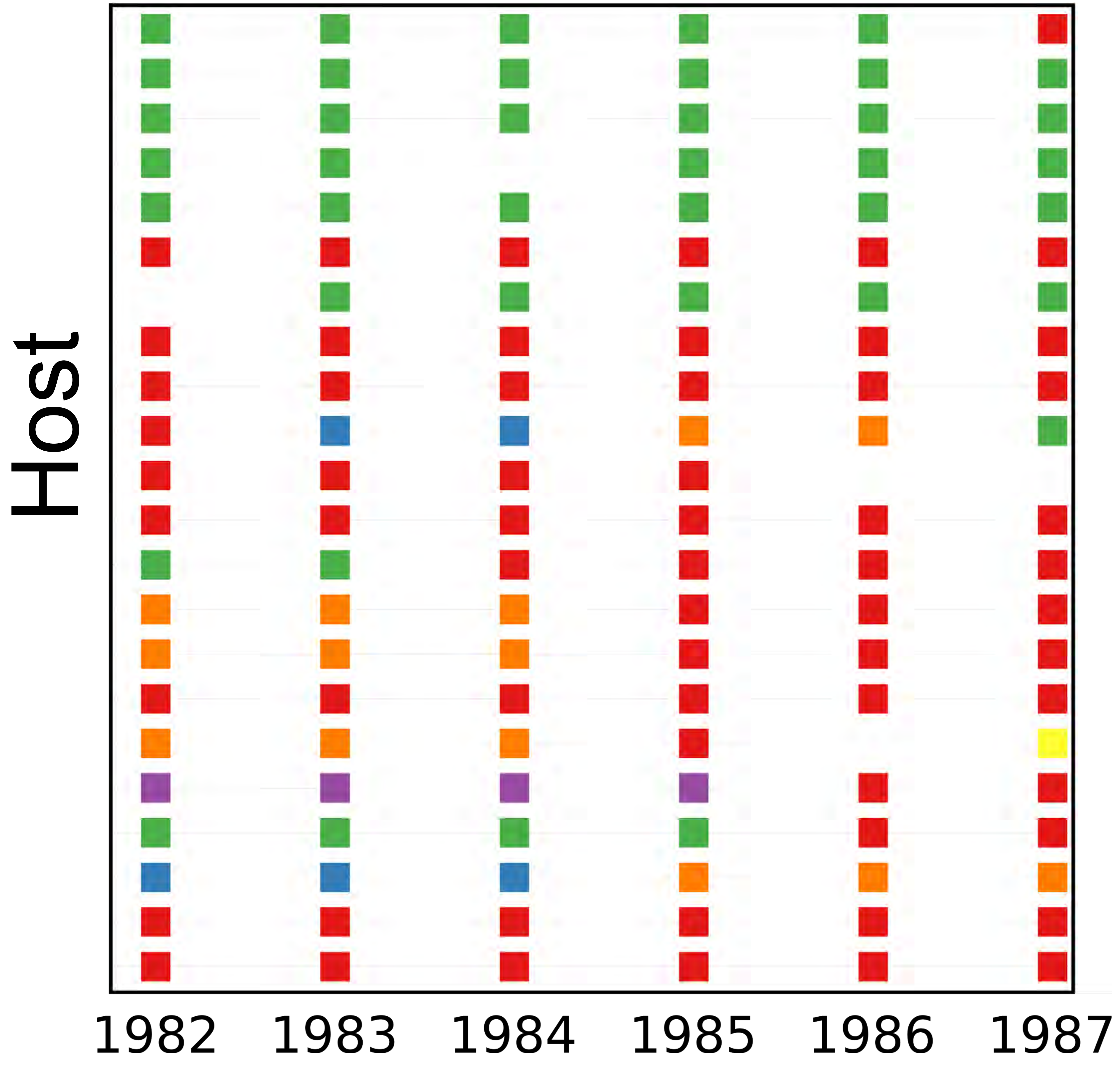
Edge weight distribution





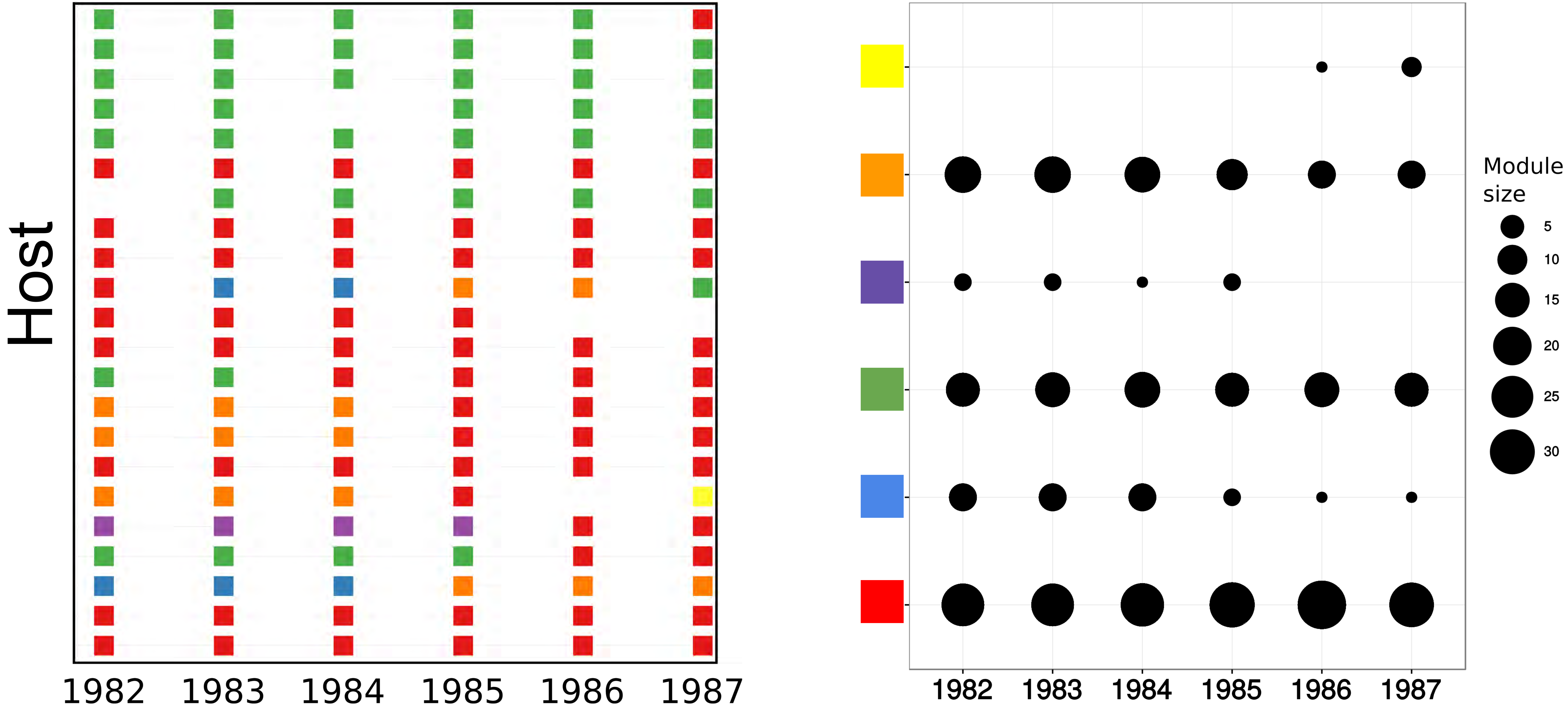
- ~6 modules with **non-random**, time-dependent structure.
- Species can change modules in time.

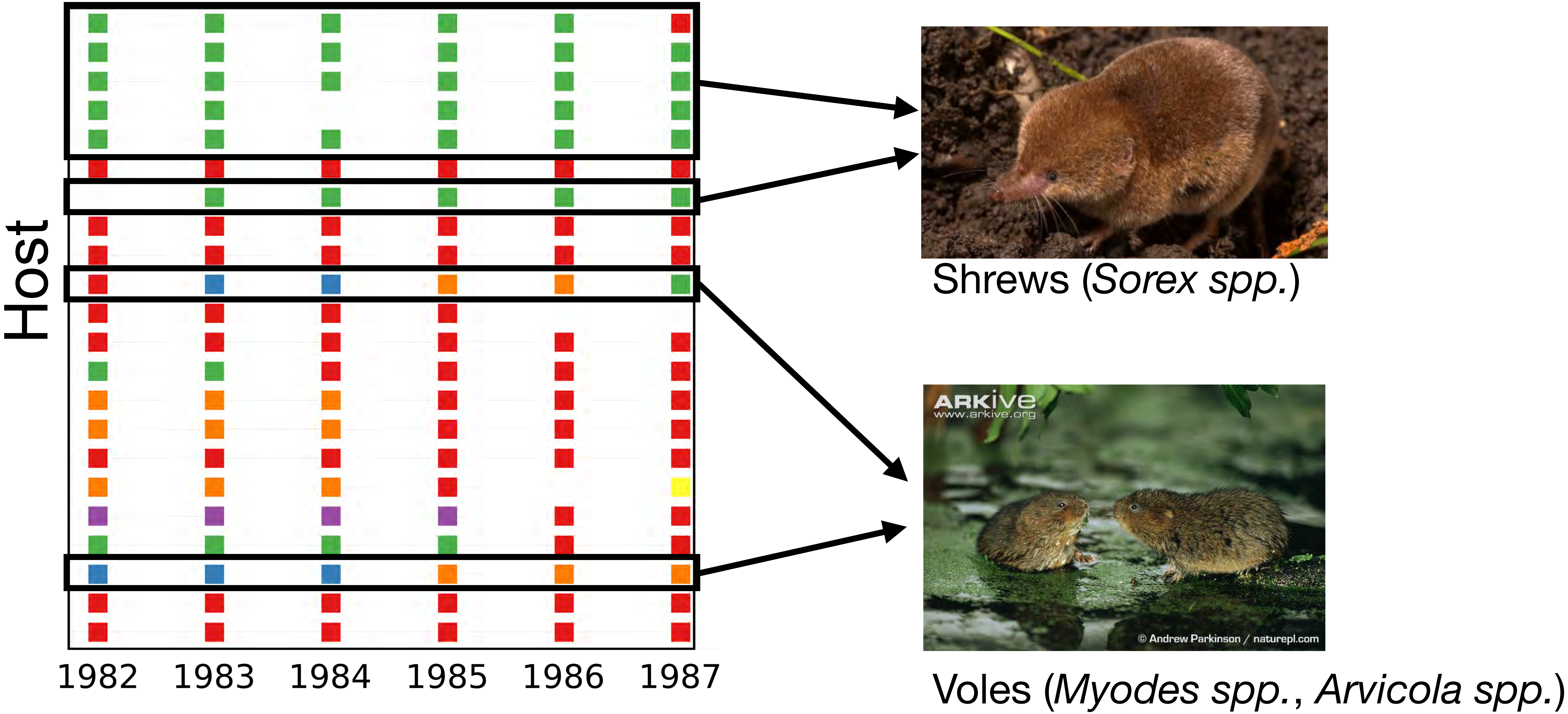
Network flexibility -- community structure evolves in time



47% hosts and 35% parasites change modules at least once

Network flexibility -- community structure evolves in time




Host
1982
1983
1984
1985
1986
1987
Shrews (*Sorex spp.*)
ARKive
www.arkive.org
© Andrew Parkinson / naturepl.com
Voles (*Myodes spp.*, *Arvicola spp.*)

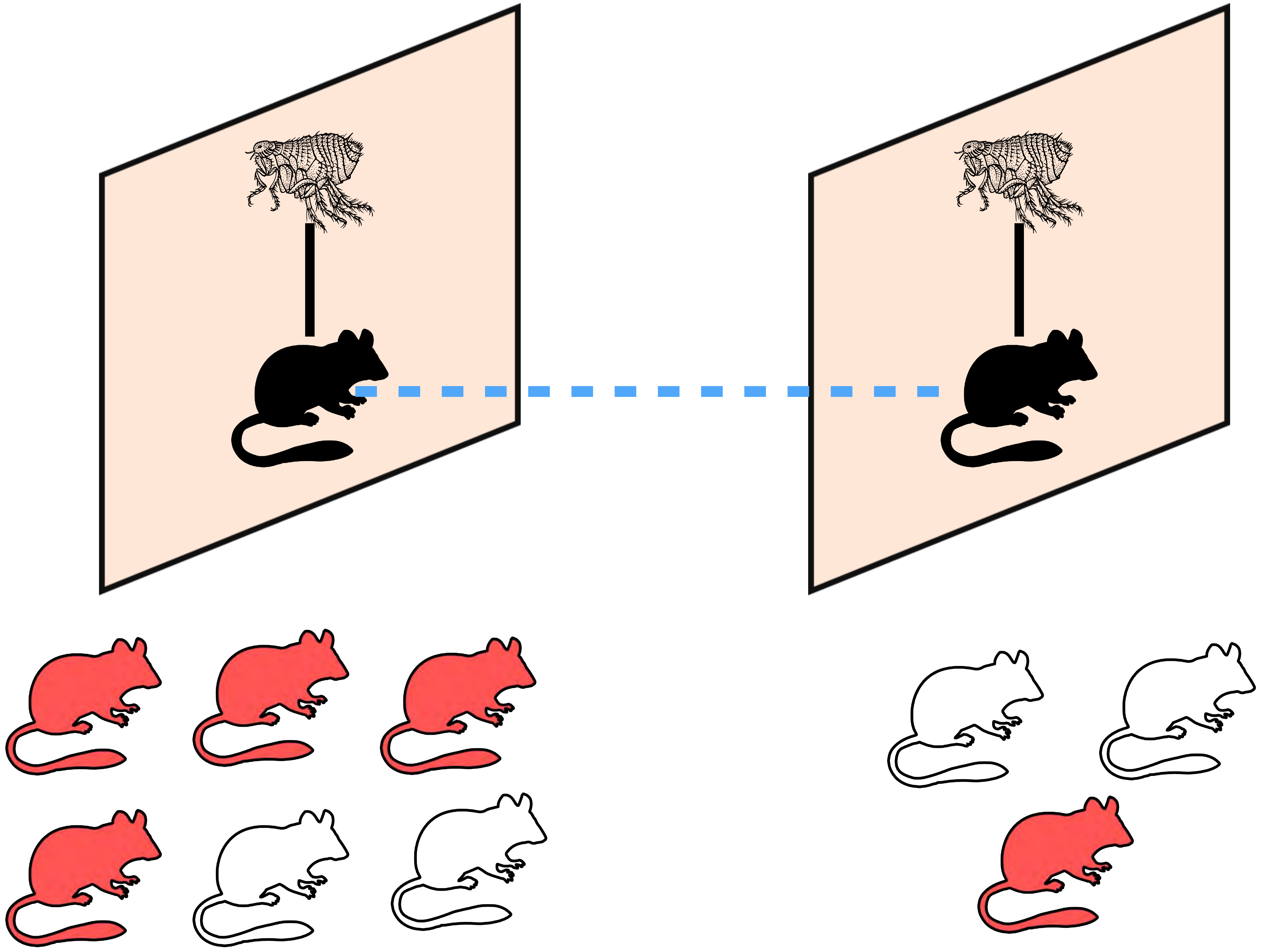
Non-flexible

- Support consistent parasite assemblages.

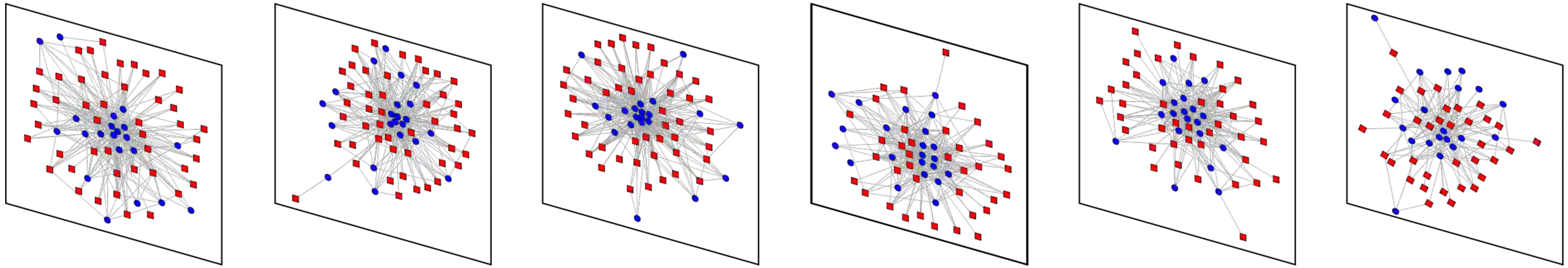
Flexible

- Support variable parasite assemblages.
- Bridge infections between species and in time.

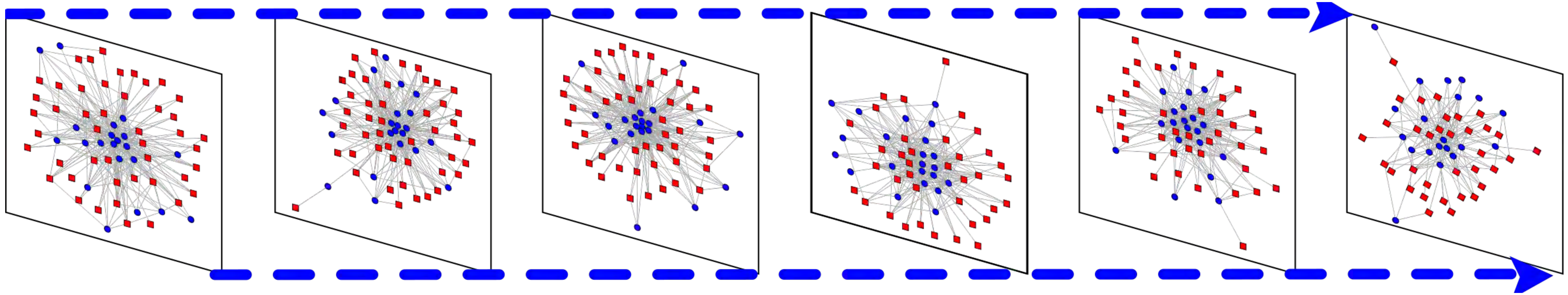
- Time-dependent distribution of parasites in hosts.
- The same species is functionally different at different times.



State nodes!



- No insights into the evolution of the modules.



- Infinity — Species do not change modules.

Interlayer edges measured in the field

$$w_{ij\alpha} = \frac{p_{ij}^{\alpha}}{b_j^{\alpha}}$$

number of bird individuals of *j* on which pollen grains of *i* was detected

Total number of bird individuals in α

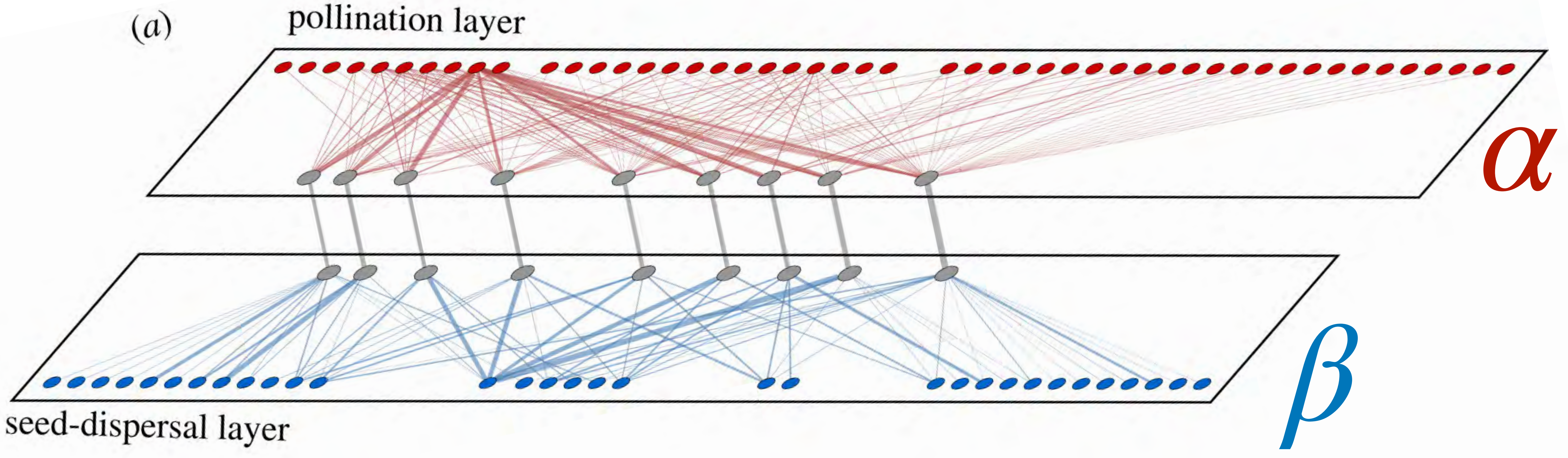
Intra (pollination)

$$w_{j\alpha\beta} = \frac{p_j^{\alpha\beta}}{b_j^{\alpha\beta}}$$

number of bird individuals of *j* on which pollen and seeds were detected

Total number of bird individuals of *j*

Interlayer (bounded [0,1])

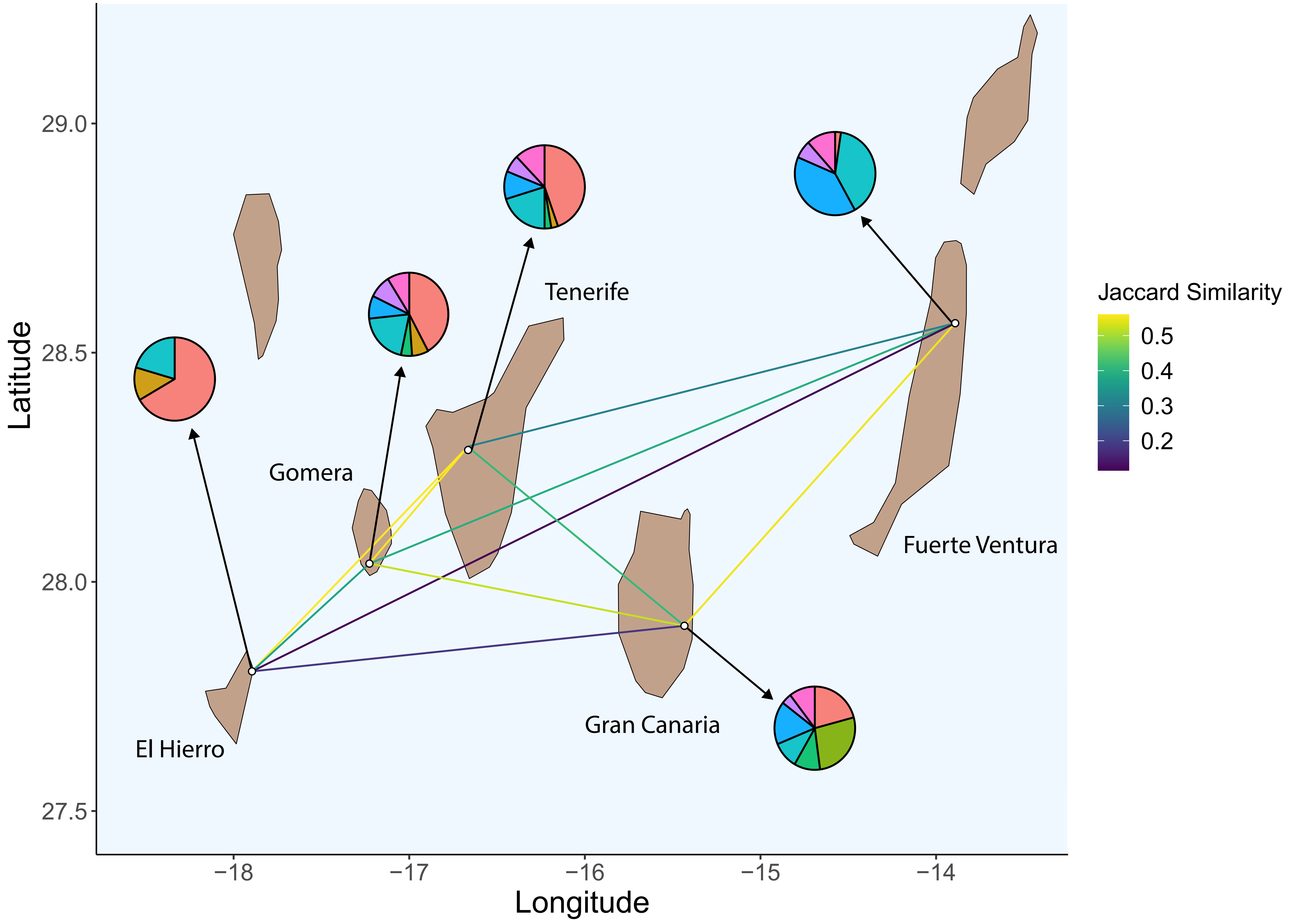


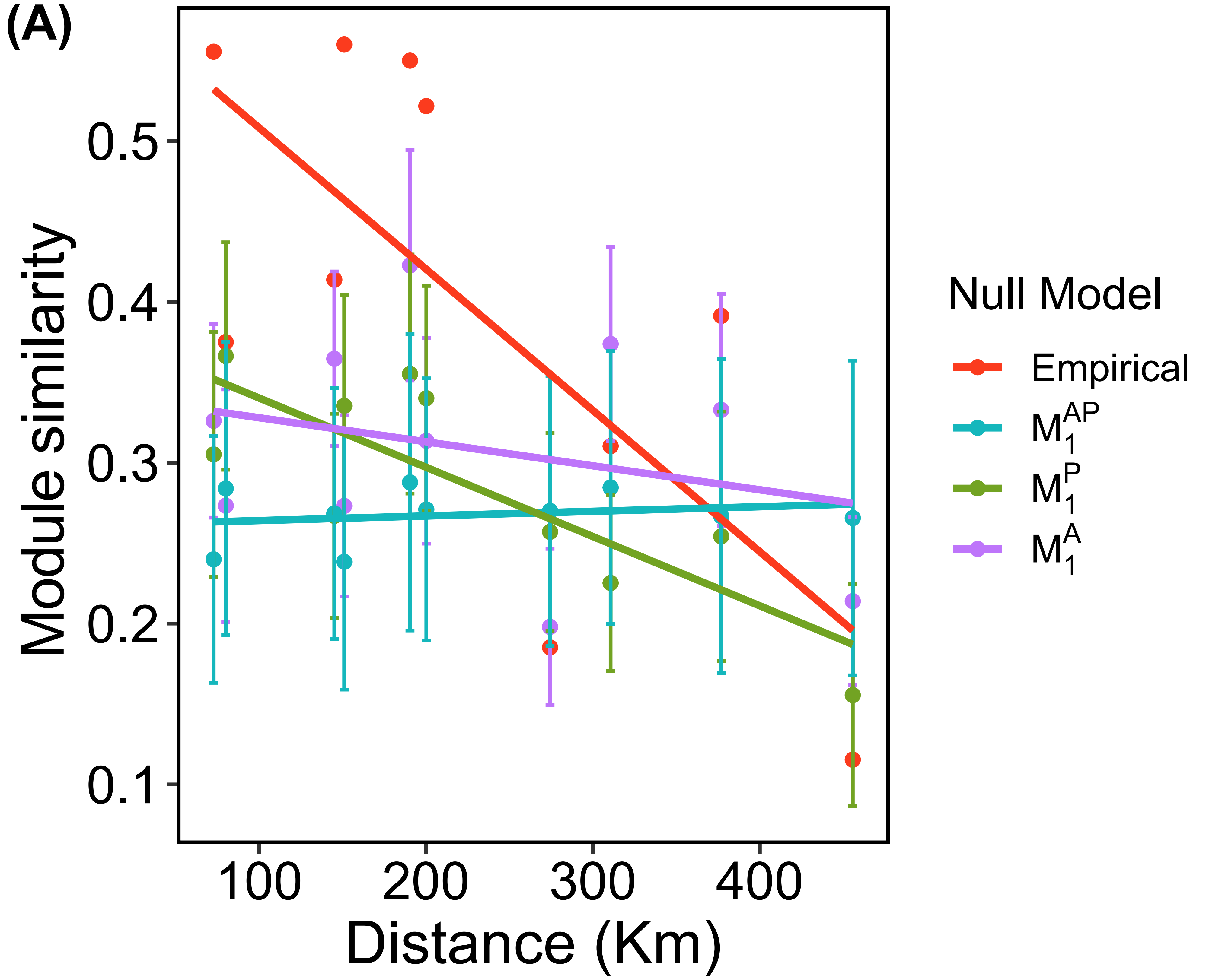
Hervías-Parejo 2020 et al., PRSLB

Example key questions

- How do multiple interaction types affect the stability of communities?
- Will a multilayer approach change understanding of ecological concepts (e.g., “key-stone species”)?
- How do processes that operate within and between layer interact to affect structure and dynamics?

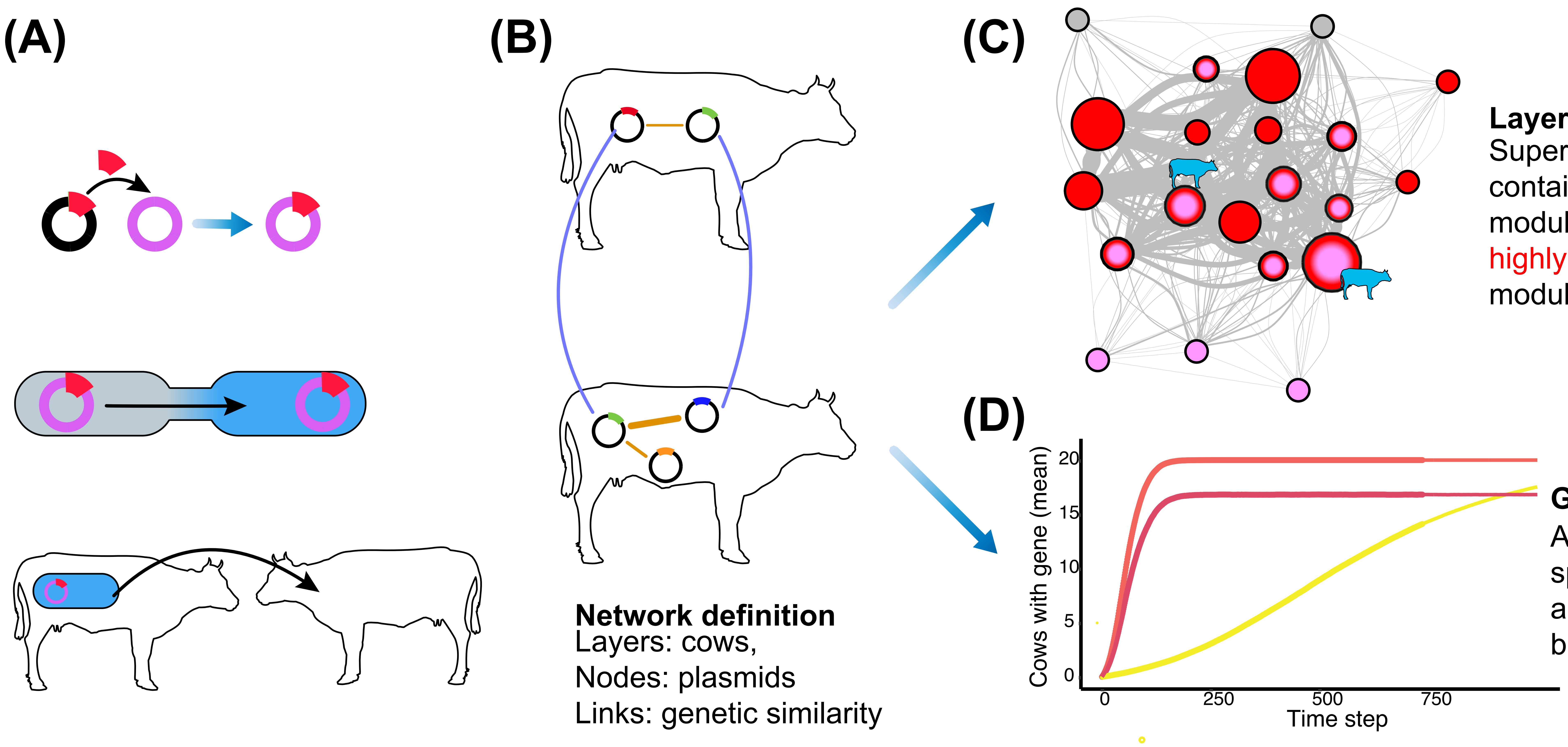
Spatial or meta-community dynamics





Vitali et al, 2024

Meta-community structure



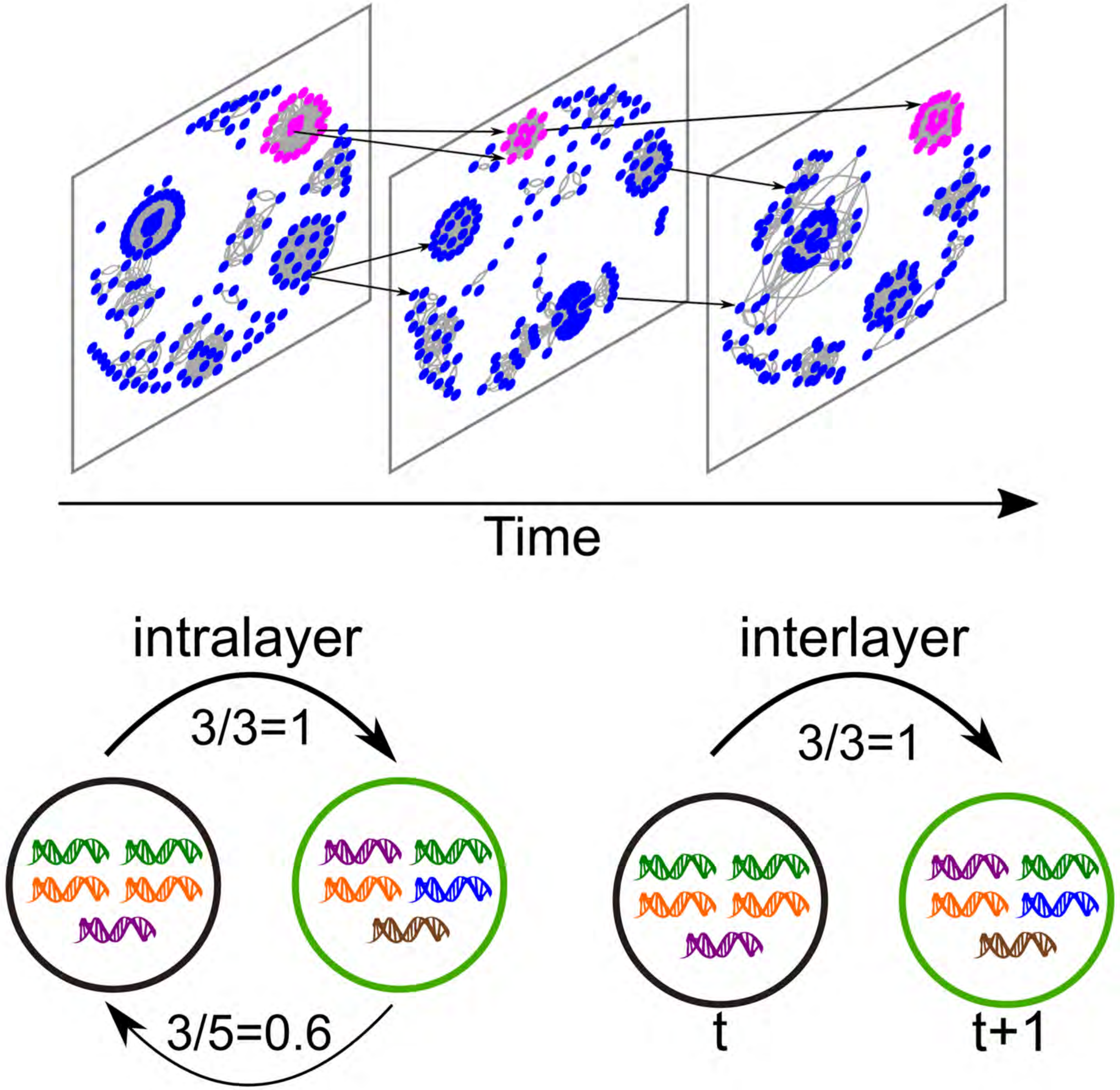
Network definition
Layers: cows,
Nodes: plasmids
Links: genetic similarity

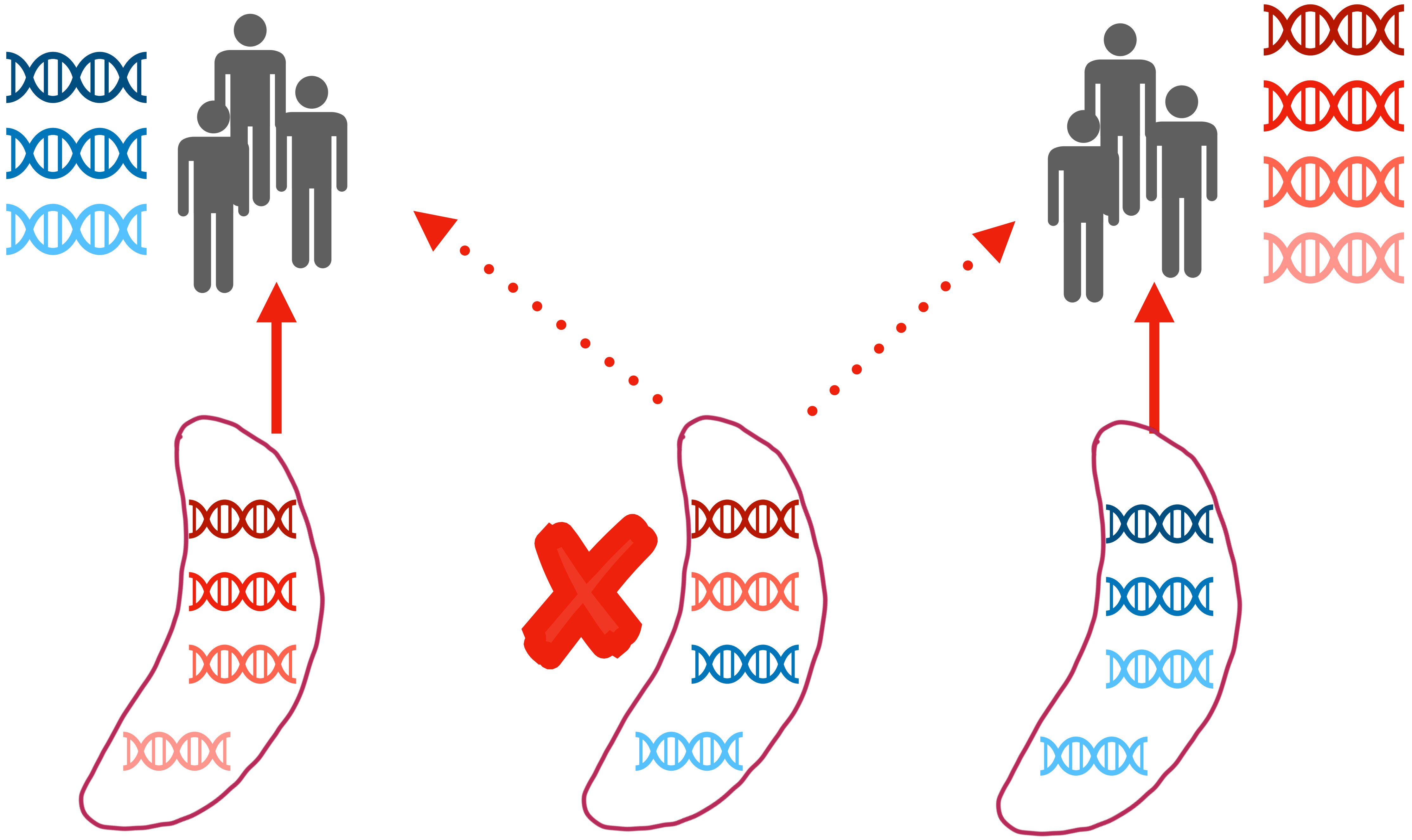
Layer-level view
Super-spreader cows contain an AMR gene module and a highly-mobile plasmid module

Gene spread
AMR genes spread quickly across plasmids & between cows.

Shapiro et al, 2023 ISME J

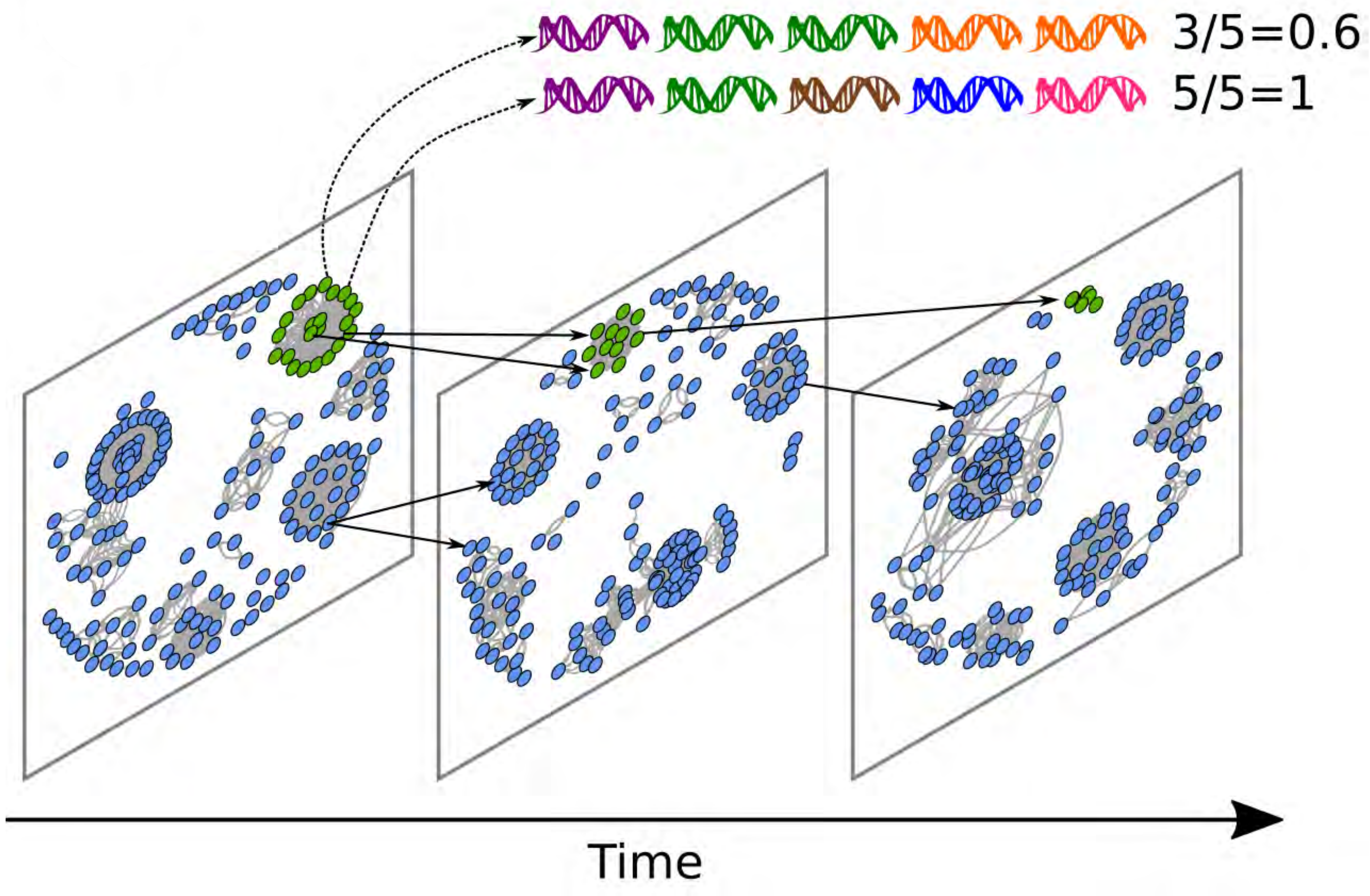
Temporal dynamics

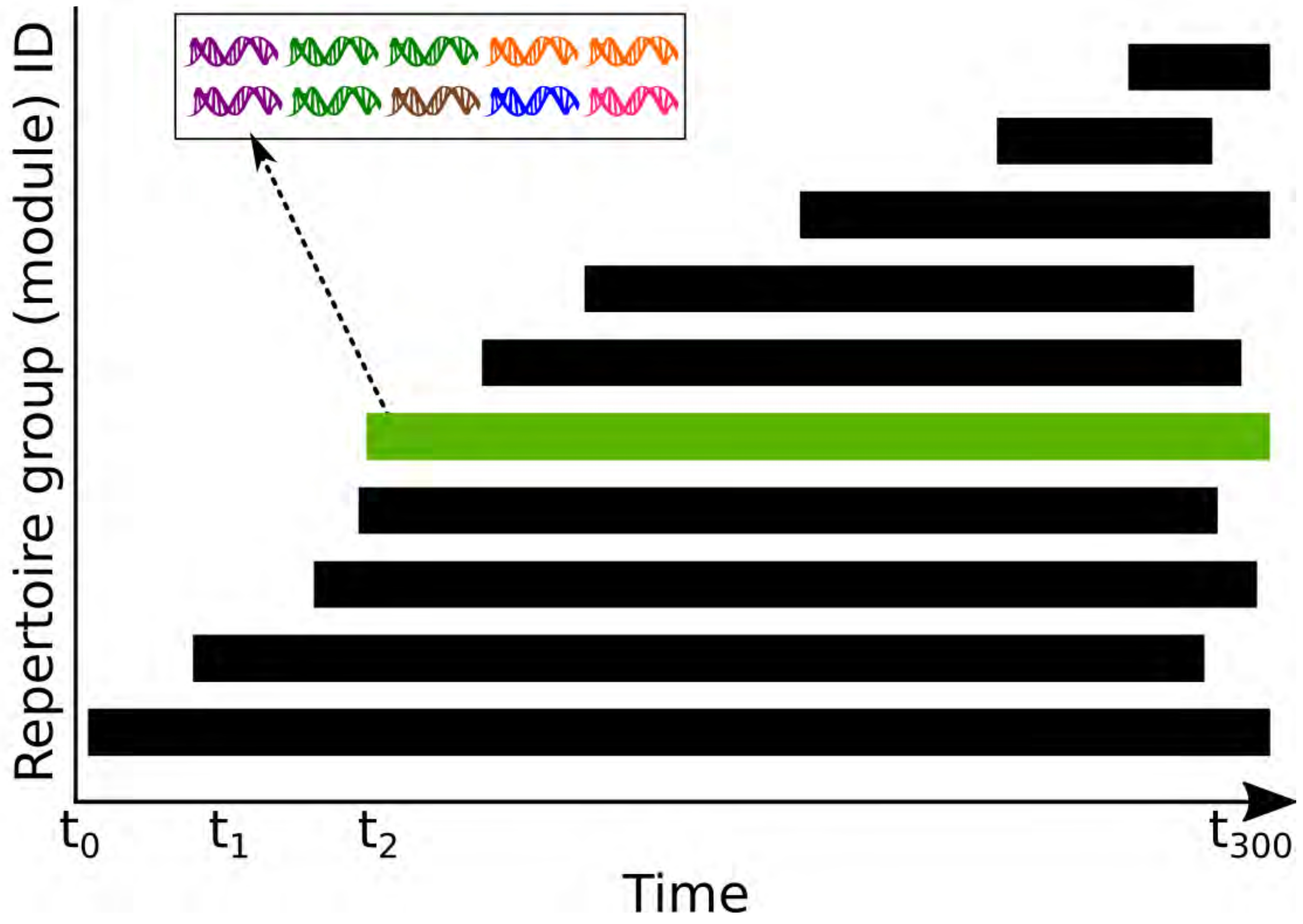




Pilosof et al 2019, PLOS Biology

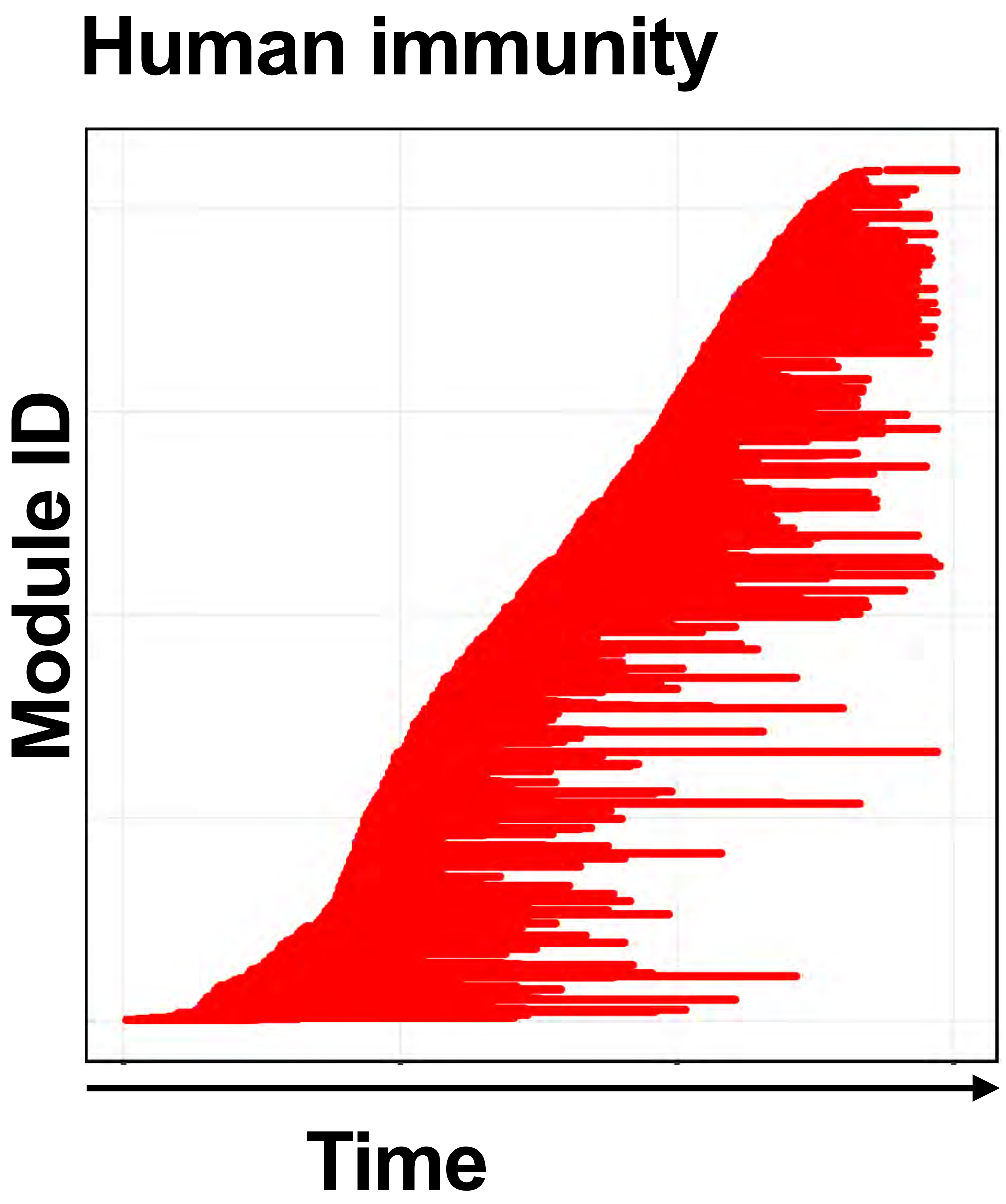
Revealing ecological niches using community detection

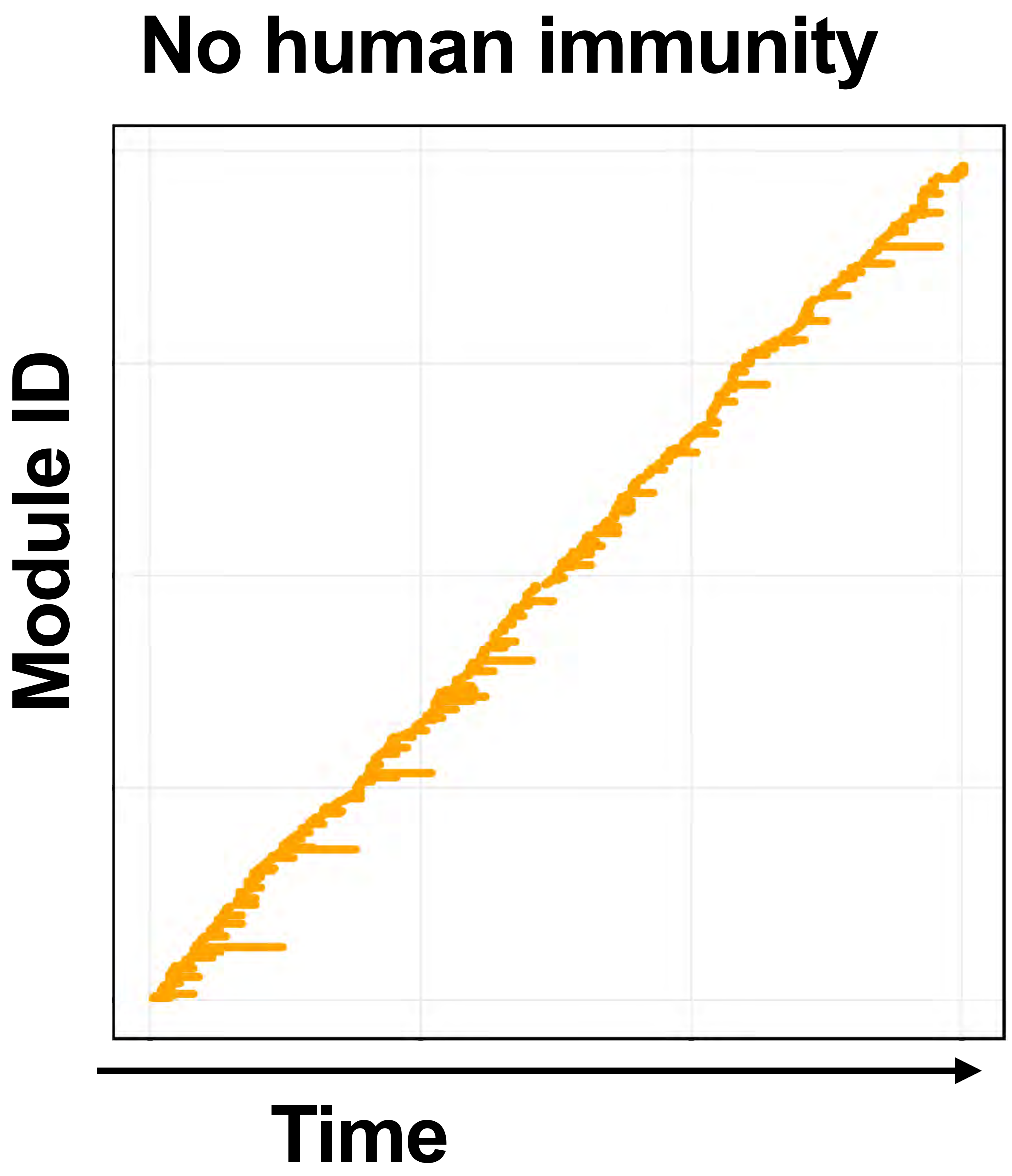




Pilosof et al 2019, PLOS Biology

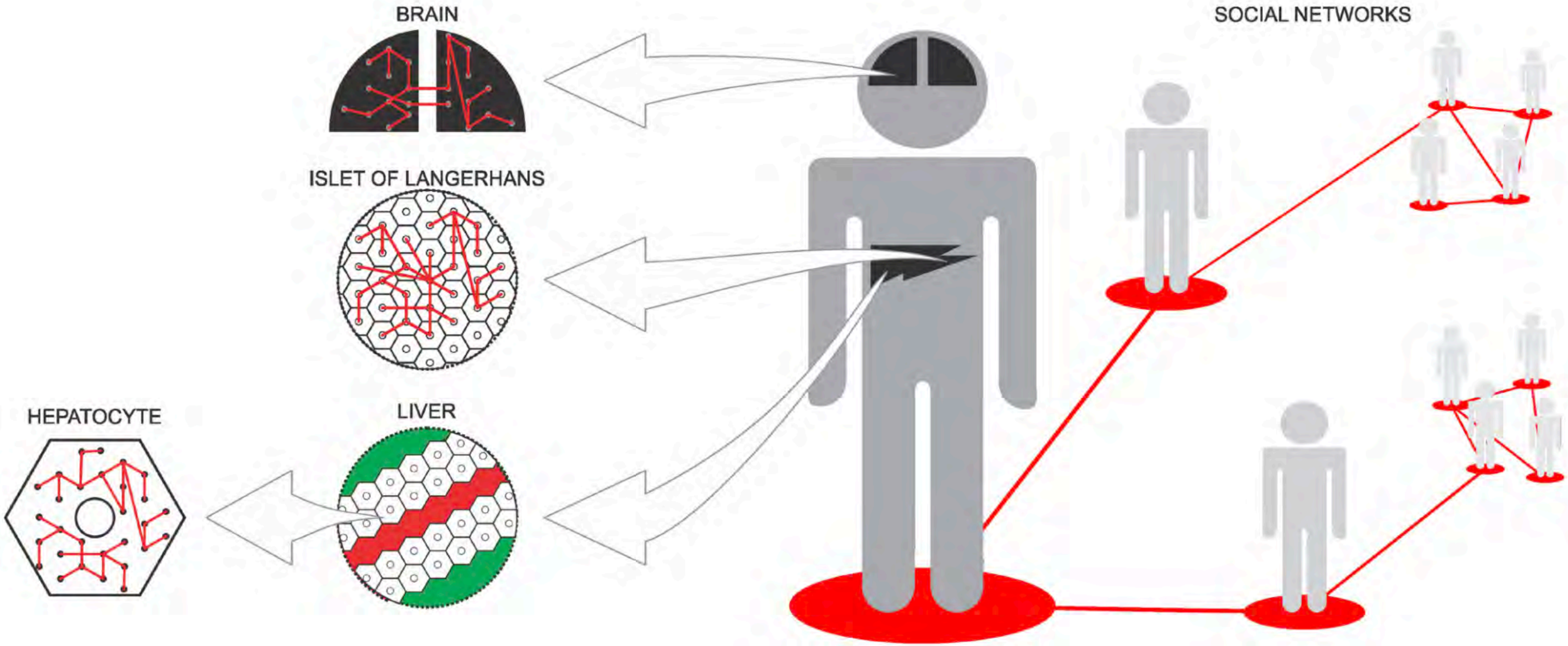
Human immunity determines structure





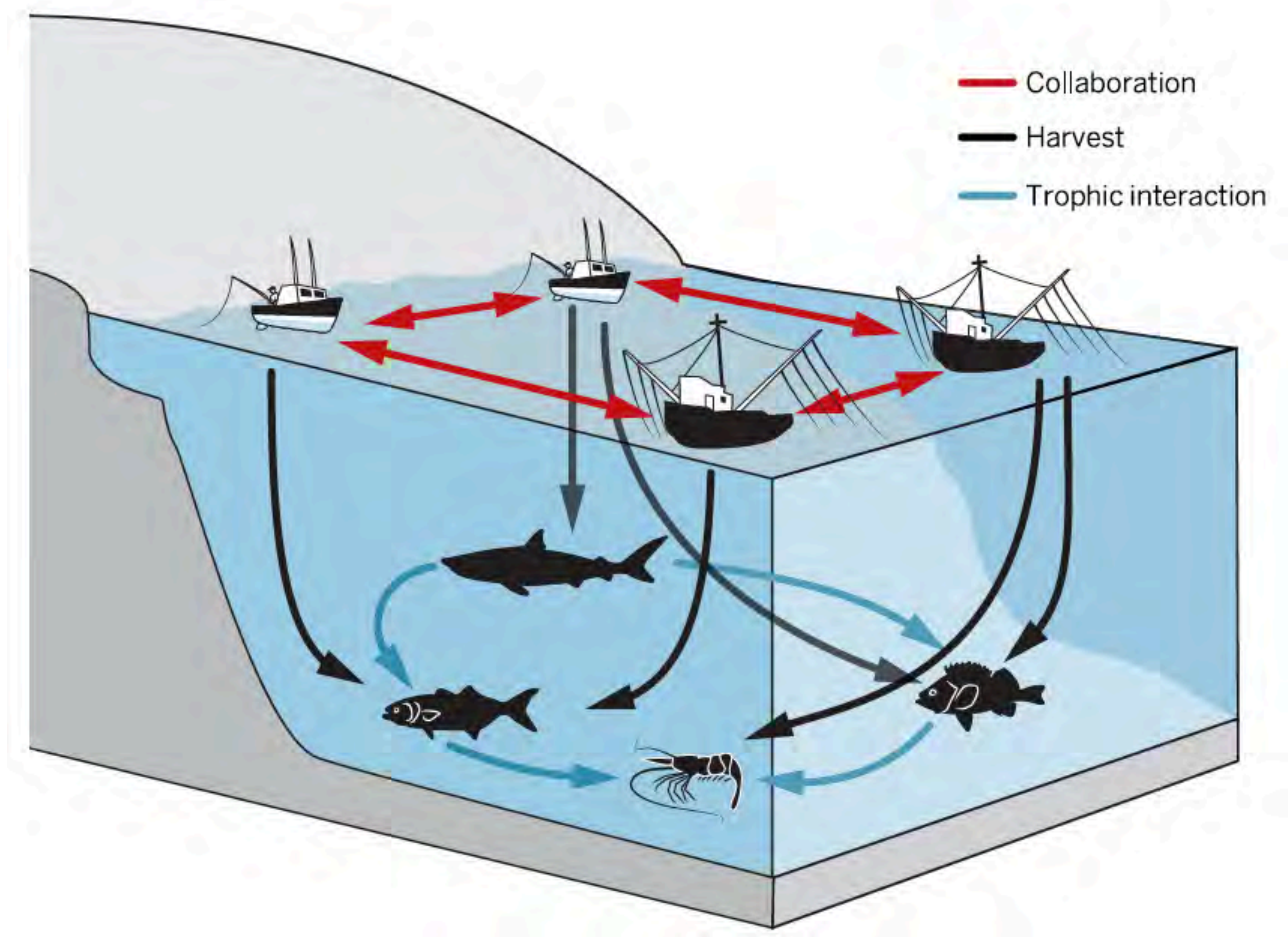
Pilosof et al 2019, PLOS Biology

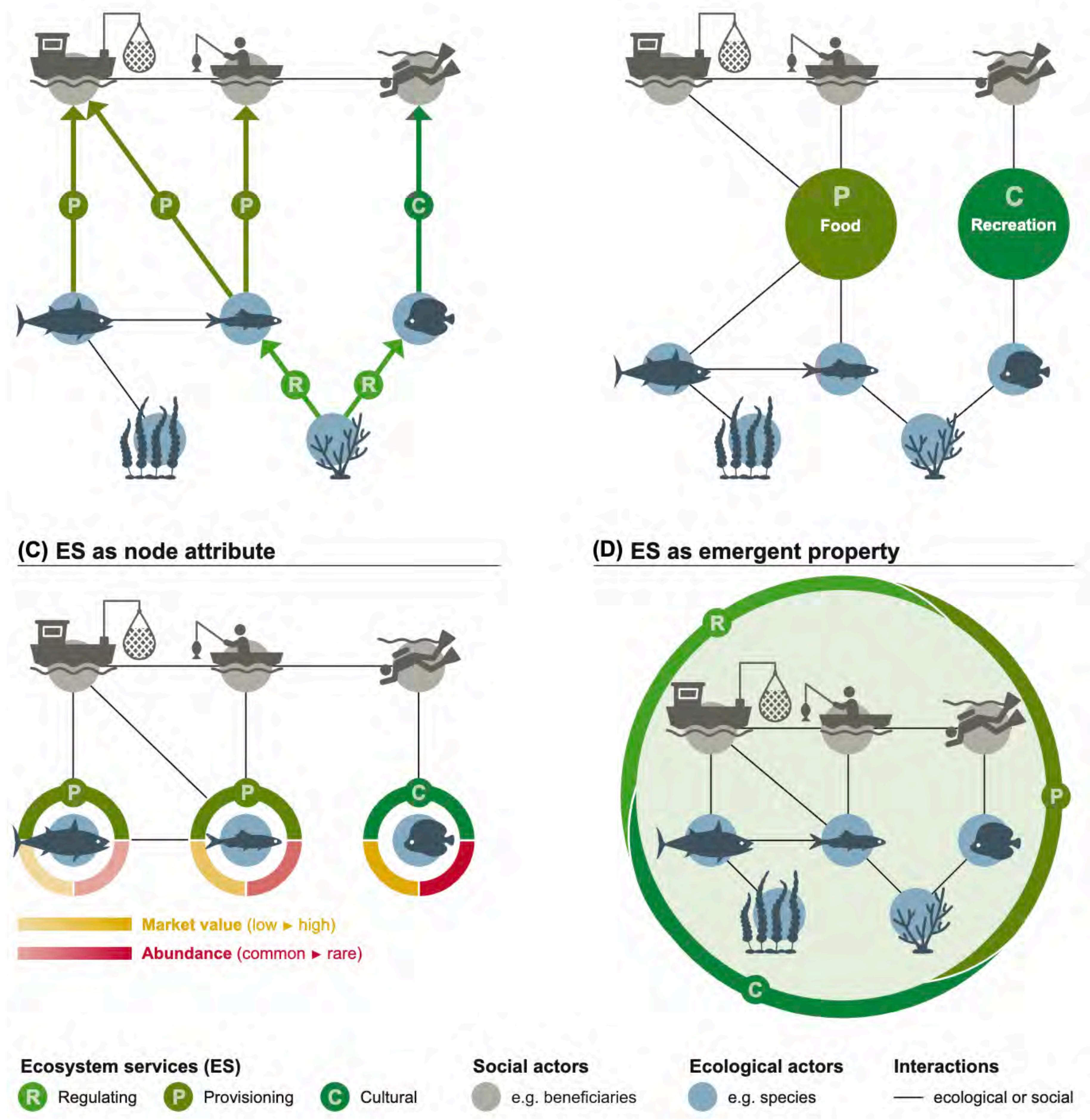
Interconnected scales



Gosak 2017, Physics of Life Reviews

Social-ecological networks





Bodin 2017 (Science)

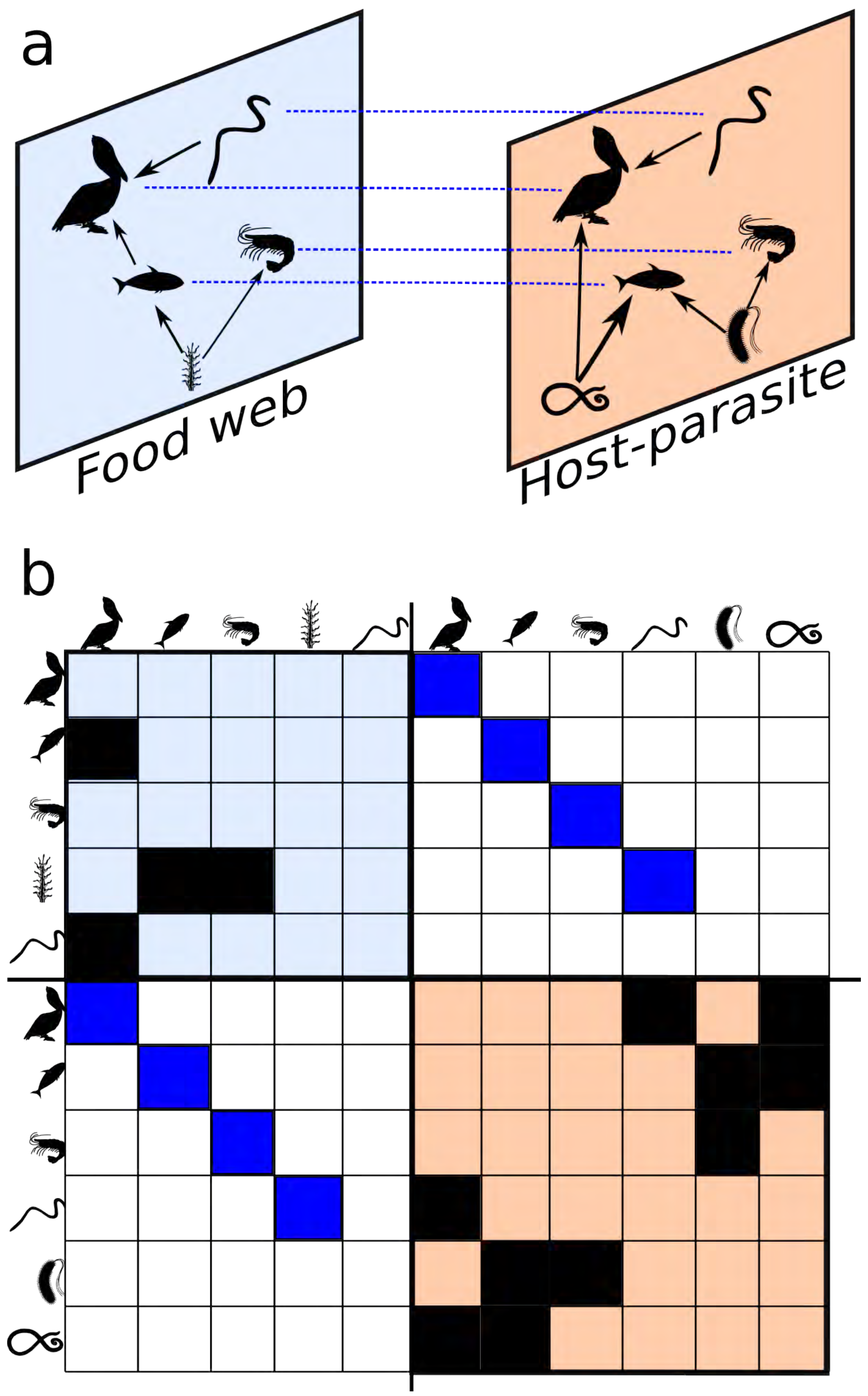
Felipe-Lucia et. al. 2022 (TREE)

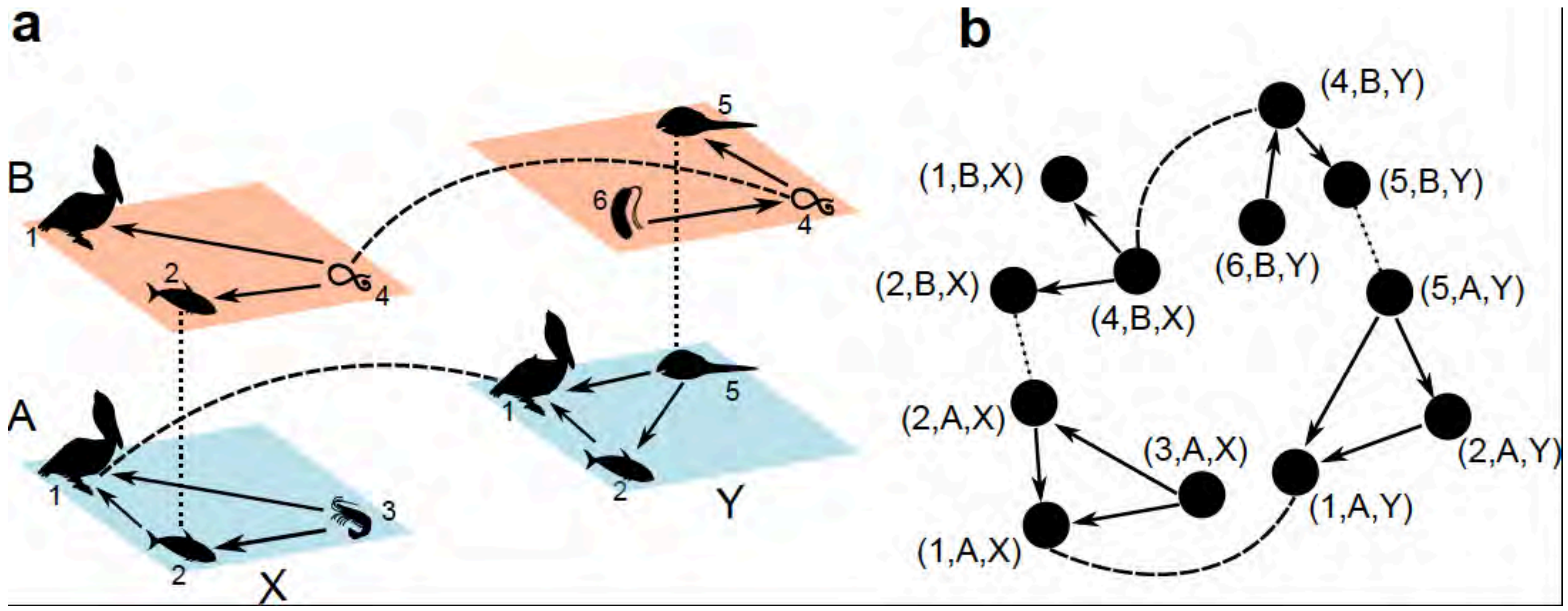
Summary

- Networks with multiple layers are not a new concept but multilayer networks is (relatively).
- Multiple types of interlayer edges.
- Interlayer edges (and state nodes) are key to encoding processes between layers.
- Many exciting questions!

Slides for exercise

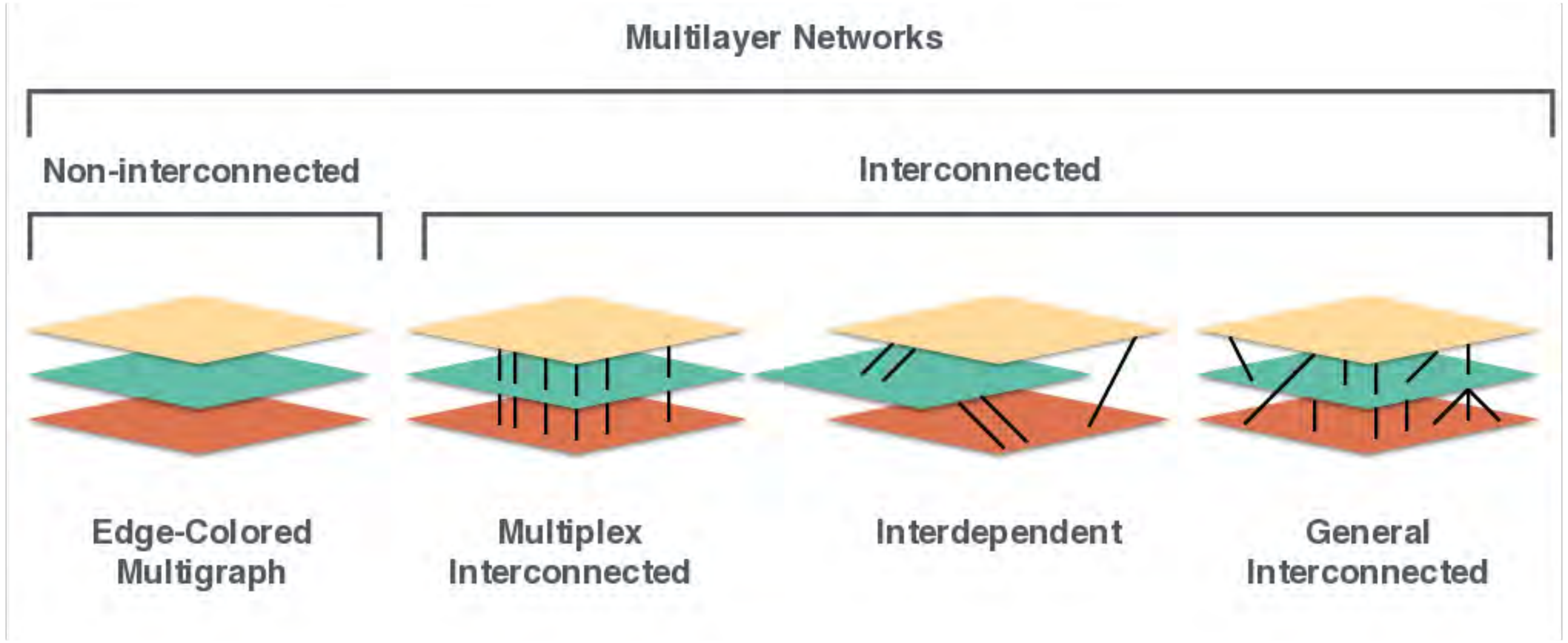
Multilayer networks can be represented as tensors (or supra-adjacency matrices)

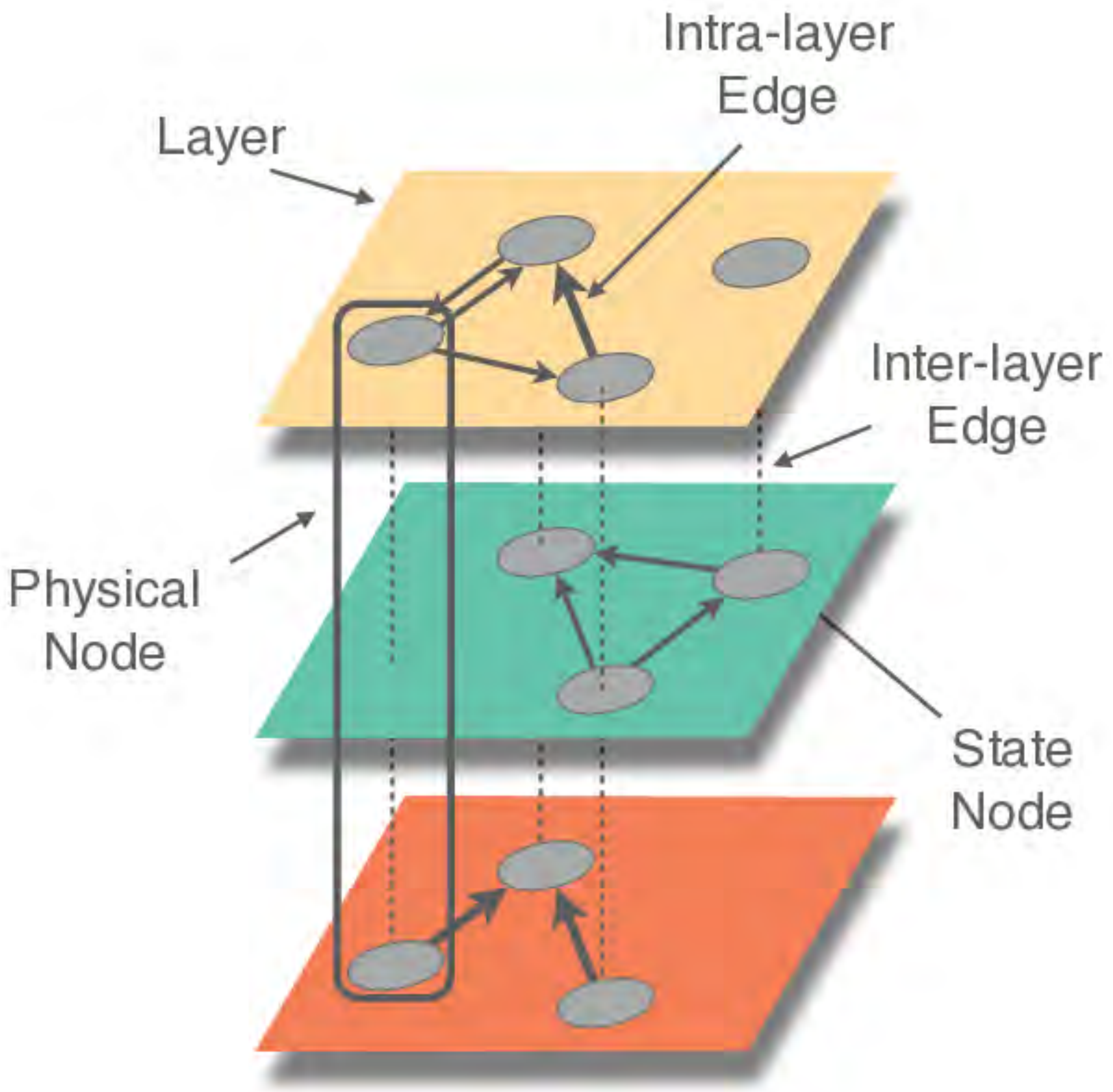




Pilosof et al. 2017, Nature Ecol. Evol.

General definitions

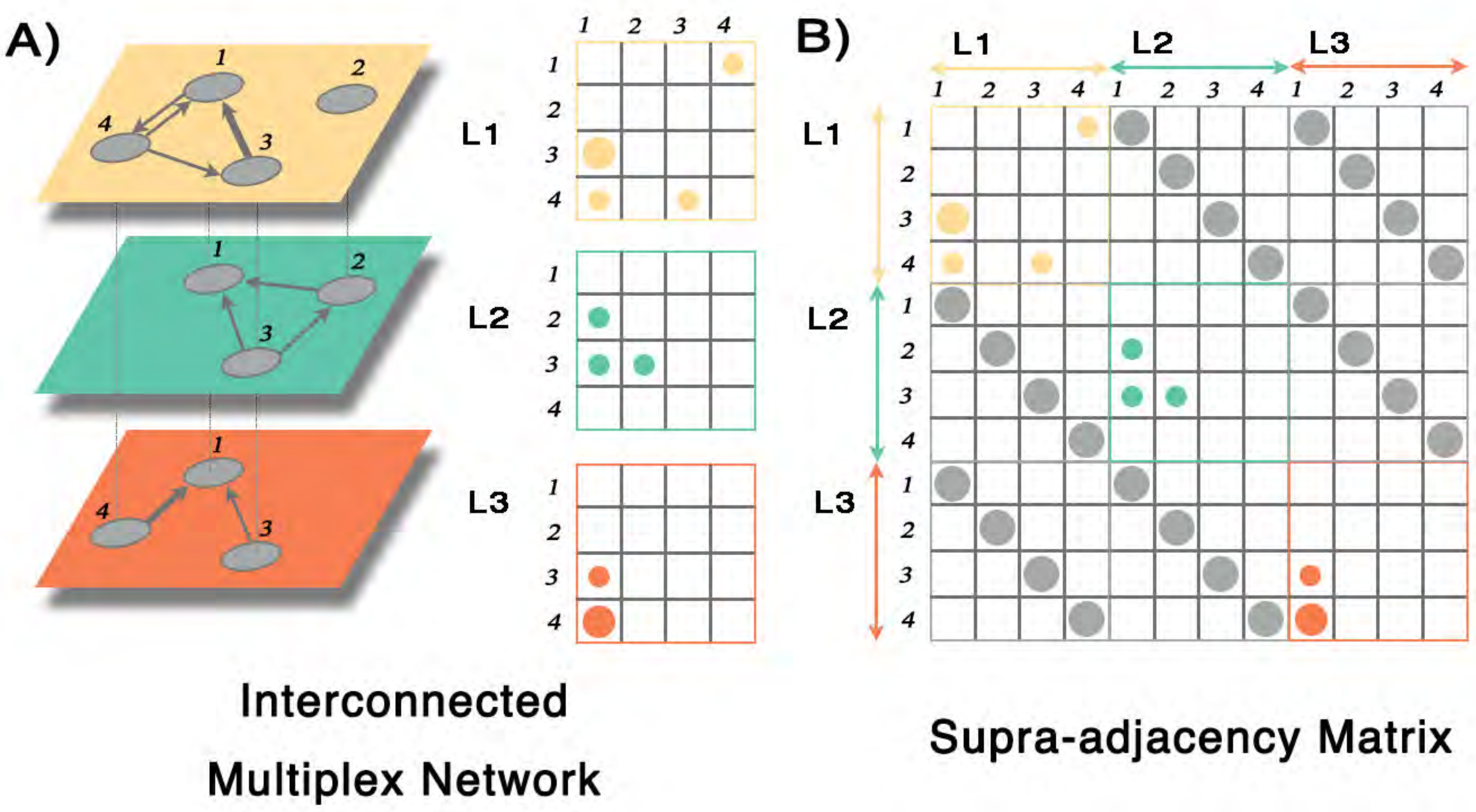




https://github.com/manlius/muxViz/blob/master/gui-old/theory/README.md

Matrix representation

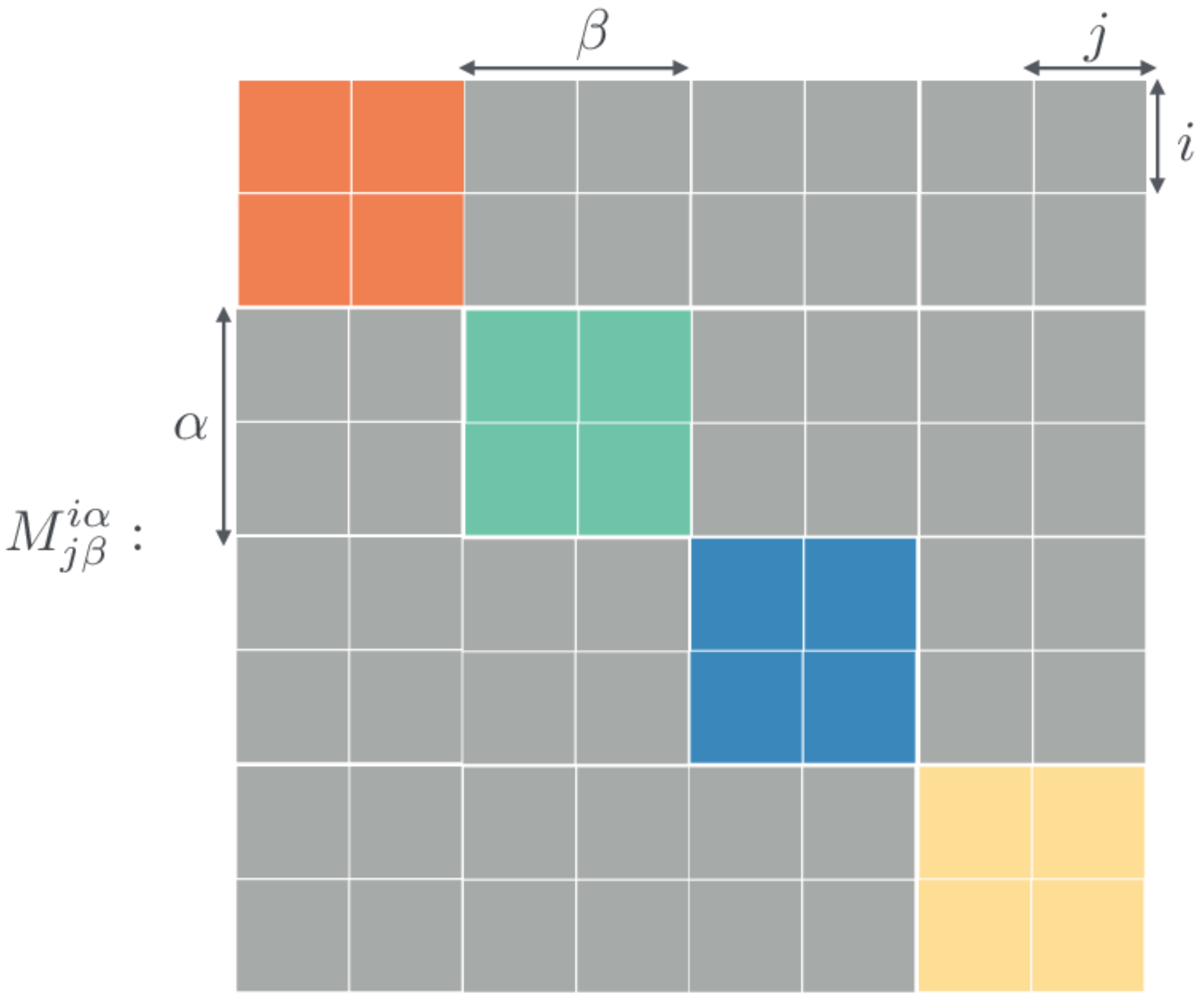
Operatively, rank-4 tensors can be mapped into rank-2 supra-adjacency matrices to facilitate operations (with care)



https://github.com/manlius/muxViz/blob/master/gui-old/theory/README.md

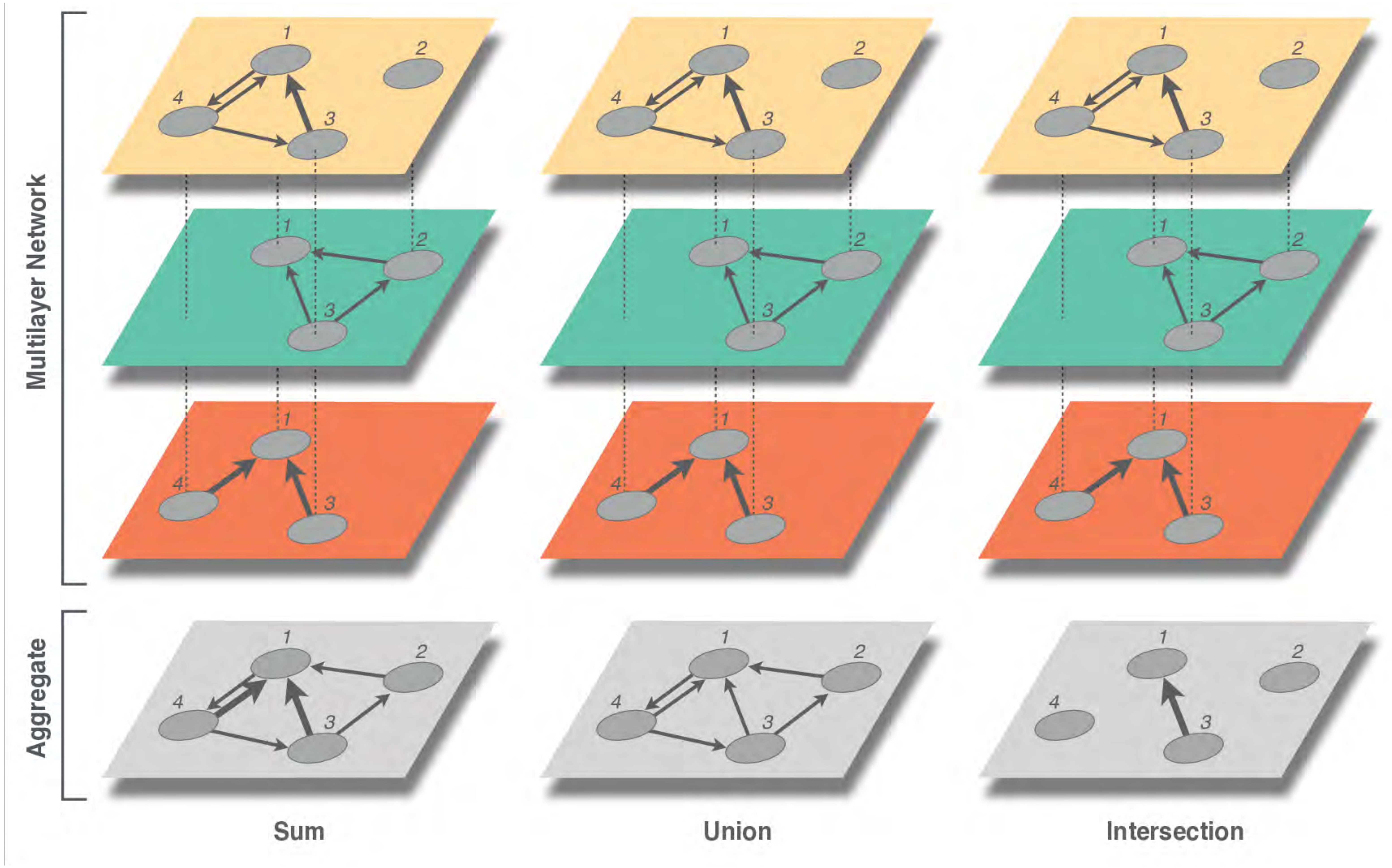
Tensor representation

A directed weighted connection between node i from layer α to node j in layer β



https://github.com/manlius/muxViz/blob/master/gui-old/theory/README.md

Aggregation



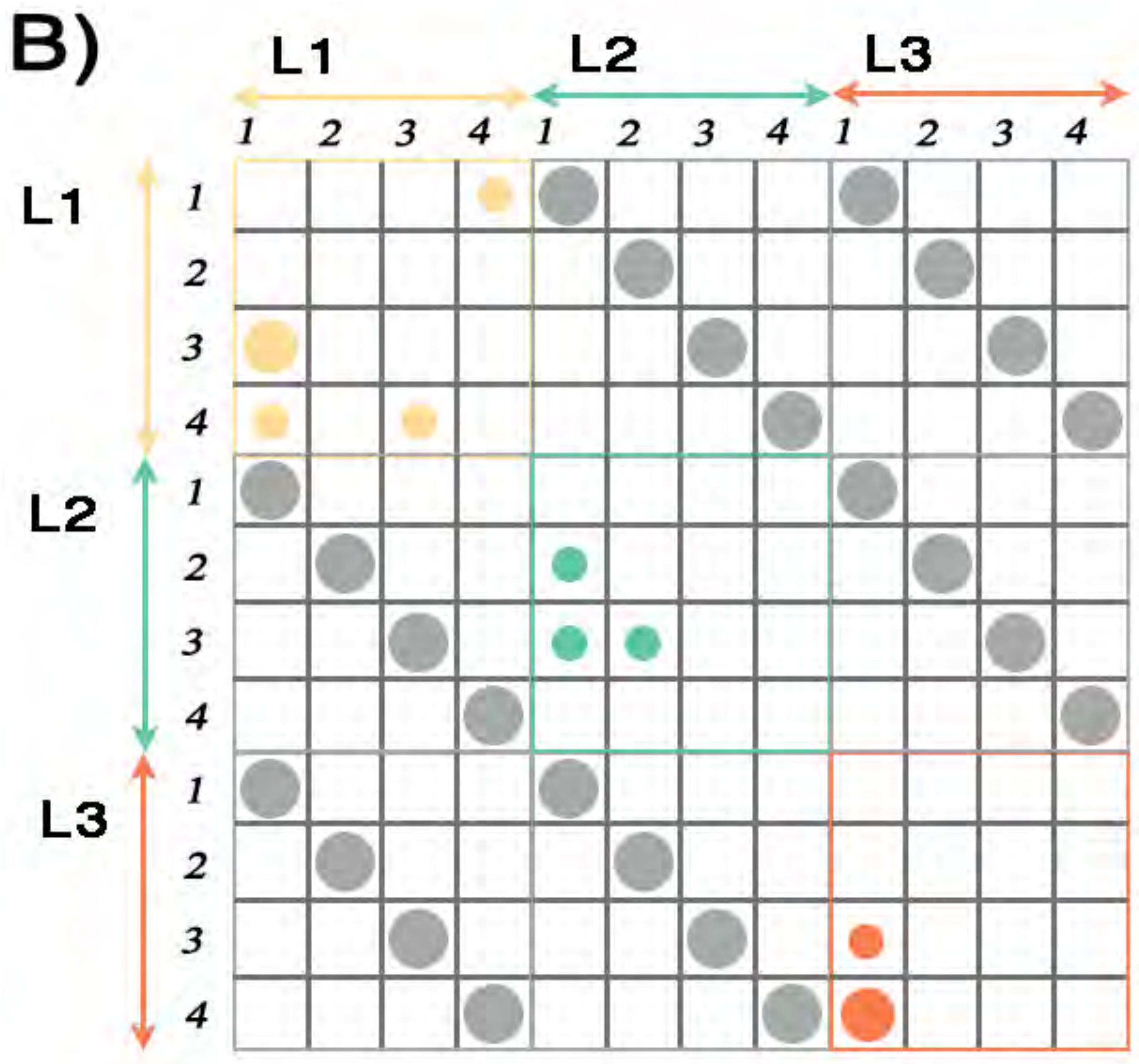
https://github.com/manlius/muxViz/blob/master/gui-old/theory/README.md

Centrality and versatility

Multi-degree of a node i is a vector (and you can do whatever you want with it…)

$$k_i = (k_i^{[1]}, \ldots, k_i^{[M]})$$

EV versatility: "The versatility of each node is obtained by aggregating over layers the centrality of each node in each layer computed using the full multilayer structure."



Bianconi 2013, PRE; De Domenico et al. Nature Comm. 2015

Centrality and versatility

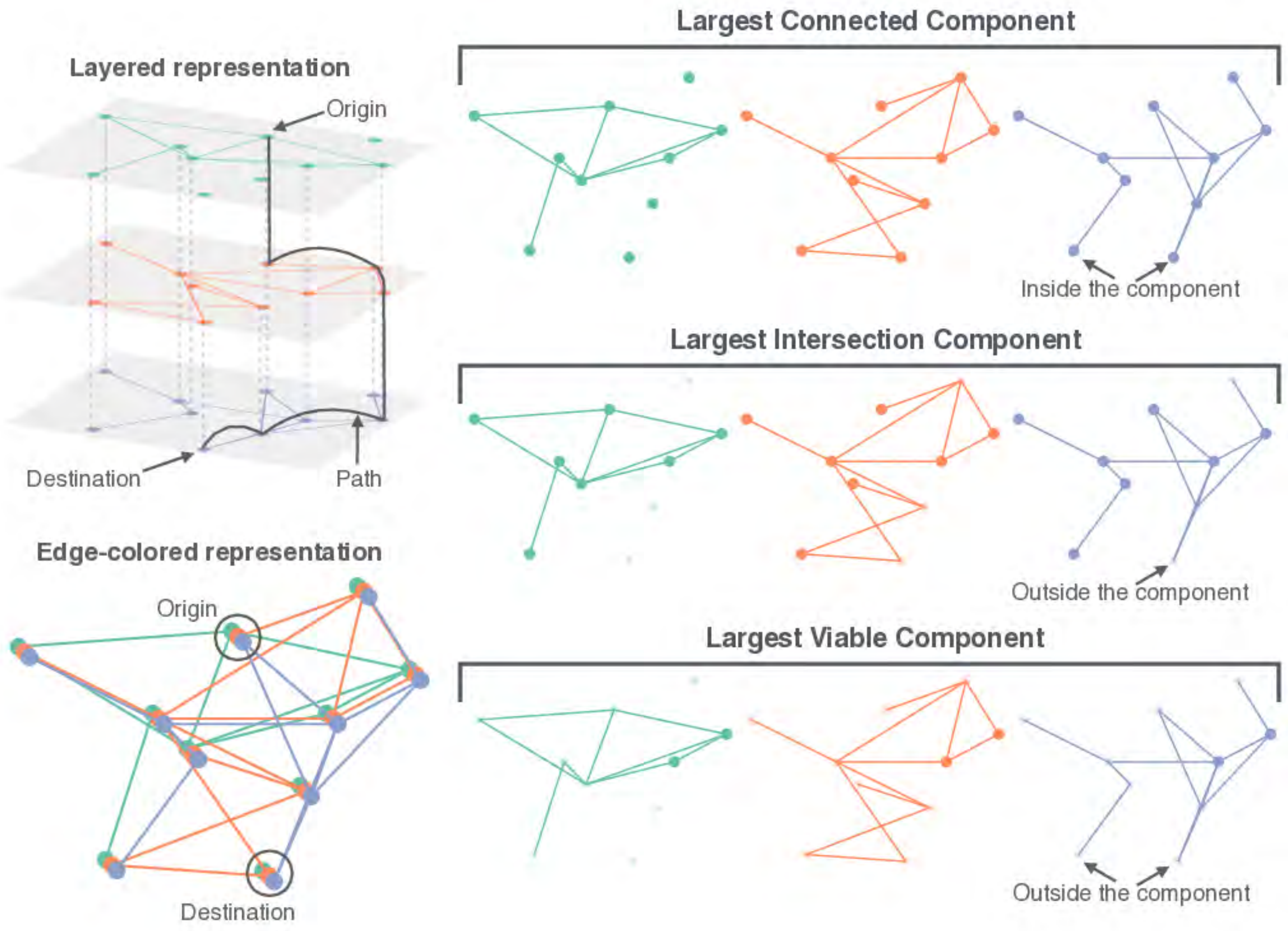
Betweenness versatility: can calculate on supra-adjacency matrix

$$l^*_{s\alpha \to t\beta} = min(\mathscr{P}_{s\alpha \to t\beta})$$

$$l^*_{s \to t} = min_{\alpha,\beta \in \{1,2,\ldots,L\}}(l^*_{s\alpha \to t\beta})$$

$$\sigma_{st}(j) = \| l^*_{s \to t}(j) \|$$

$$BC(j) = \sum_{s \neq j \neq t} \frac{\sigma_{st}(j)}{\sigma_{st}}$$

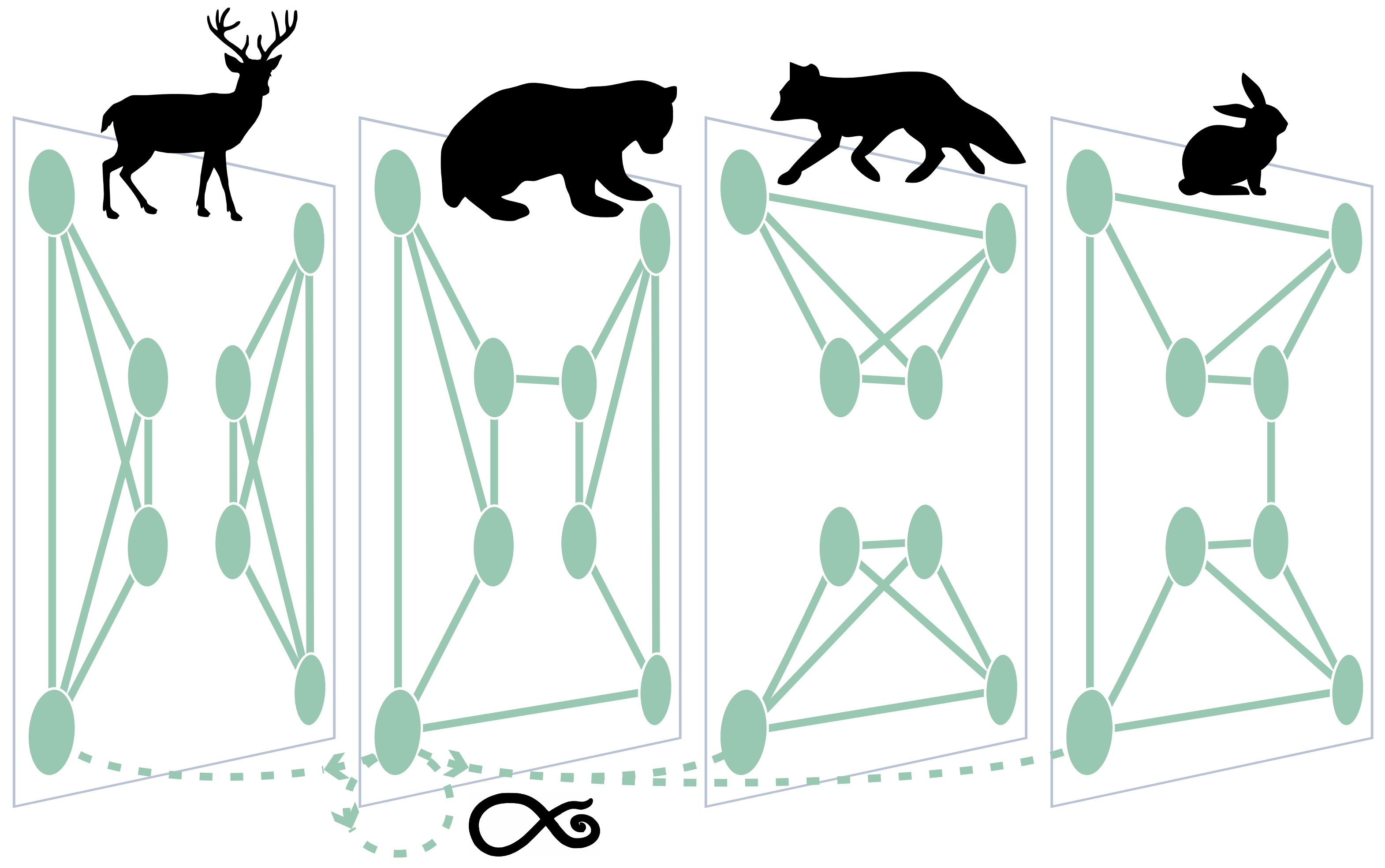


De Domenico et al. Nature Comm. 2015

Infomap in multilayer networks

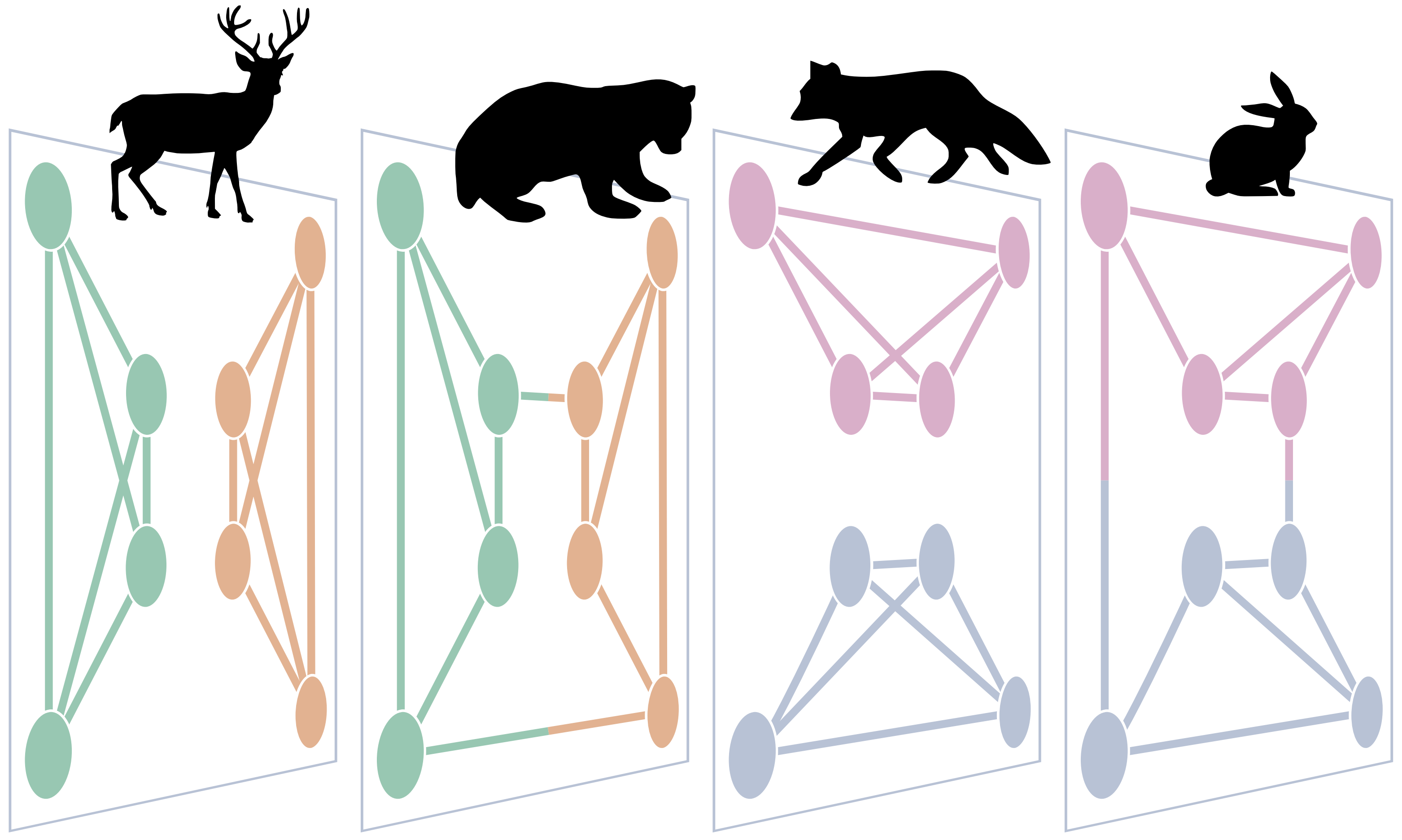
https://www.mapequation.org/apps/multilayer-network/index.html

(a)



$$L = H(12/96, 12/96, 12/96, 12/96, 12/96, 12/96, 12/96, 12/96) = 3 \text{ bits}$$

(b)



$$L = (8 + 48r)/96H((2 + 12r)/96, (2 + 12r)/96, (2 + 12r)/96, (2 + 12r)/96)+$$
$$(26 + 12r)/96H(6/96, 6/96, 6/96, 6/96, (2 + 12r)/96)+$$
$$(26 + 12r)/96H(6/96, 6/96, 6/96, 6/96, (2 + 12r)/96)+$$
$$(26 + 12r)/96H(6/96, 6/96, 6/96, 6/96, (2 + 12r)/96)+$$
$$(26 + 12r)/96H(6/96, 6/96, 6/96, 6/96, (2 + 12r)/96)+$$
$$\approx 2.84 \text{ bits for } r = 0.1$$

Farage, Edler, Eklöf, Rosvall, Pilosof 2021 Methods Ecol Evol.